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第1章 基礎解析

1.1 式の計算

$$1.1 \quad (1) \frac{3}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} \qquad (2) \frac{2}{(x^2+1)} - \frac{1}{x+1} + \frac{1}{x-1}$$

$$(3) \sum_{k=0}^n \frac{(-1)^k}{k!} (n-k)! (x+k)$$

$$1.2 \quad 1, 2, -3$$

$$1.3 \quad \lambda_1 = \frac{3}{2}, \lambda_2 = \frac{2}{3}b - \lambda, \lambda_3 = \lambda, \lambda_4 = c - \frac{2}{3}b\lambda + \lambda^2 \quad \text{ただし、}\lambda \text{は任意の数でしかも } 12a = 8b + 9 \\ \text{が成り立っていなければならない。それ以外は解なし。}$$

$$1.4 \quad \alpha^2 \neq \beta \text{ のとき、 } a = \frac{m - p\alpha - q}{\alpha^2 - \beta}, b = p, c = \frac{p\alpha\beta + q\alpha^2 - m\beta}{\alpha^2 - \beta}, \quad m \neq p\alpha + q \text{ のとき、不能}$$

$$1.5 \quad \frac{r(\sqrt{h^2 + r^2} - r)}{h}$$

1.2 方程式

$$2.1 \quad x = 3 \pm 2i, 4 \pm \sqrt{3}$$

$$2.2 \quad \text{略}$$

$$2.3 \quad (1)x^3 + (2q - p^2)x^2 + (q^2 - 2pr)x - r^2 = 0 \qquad (2)x^3 - qx^2 + prx - r^2 = 0$$

$$2.4 \quad x = t, y = 2t - 1, z = t \quad \text{ただし } t \text{ は任意の実数}$$

$$2.5 \quad \text{略, } a_{n+3} + pa_{n+2} + qa_{n+1} + ra_n = 0 \text{ が成立}$$

$$2.6 \quad \sqrt[3]{3}, \sqrt[3]{3}\omega, \sqrt[3]{3}\omega^2, \omega = \frac{-1 \pm \sqrt{3}i}{2}$$

1.3 三角関数

$$3.1 \quad 8x^4 - 8x^2 + 1$$

$$3.2 \quad x = n\pi - \frac{\pi}{4}, n\pi + \tan^{-1}\sqrt{6} - 2 \pm (\sqrt{3} - \sqrt{2}) \quad (n \text{ は任意の整数})$$

$$3.3 \quad 0 < \gamma < \pi \text{ のとき、 } 2\sin\frac{\gamma}{2}, \alpha = \frac{\gamma}{2} + 2n\pi, \beta = \frac{\gamma}{2} - 2n\pi \\ -\pi < \gamma < 0 \text{ のとき、 } -2\sin\frac{\gamma}{2}, \alpha = \frac{\gamma}{2} + 2n\pi + \pi, \beta = \frac{\gamma}{2} - 2n\pi + \pi$$

$$3.4 \quad \frac{\sqrt{2}-1}{2\cos x} + \frac{\sqrt{2}-\sqrt{3}}{2\sin x}$$

3.5 (1) ± 35 (2) $\frac{4}{3}, \frac{3}{4}$ (3) $-2, 3, \frac{1}{2}, -\frac{1}{3}$

3.6 $a = 5, \tan \phi = \frac{3}{4}$

1.4 領域

4.1 図略

4.2 $(-3, -1)$ で最大値 1 , $(2, -2)$ で最小値 -6

1.5 場合の数

5.1 (1) 50 (2) 67 (3) 26

5.2 9

5.3 600

5.4 (1) 3432 (2) $f(i, j) \times f(m - i, n - j)$
 (3) $f(i, j) \times f(m - i, n - j) + f(k, l) \times f(m - k, n - l) - f(i, j) \times f(k - i, l - j) \times f(m - k, n - l)$

5.5 $00 \quad 01 \quad 10 \quad 11, \quad 2040 \quad 4 \text{ ビット} \quad 5 \text{ ビット}$

5.6 25200

5.7 2^n

5.8 56

1.6 二項定理

6.1 (1) 1 (2) x (3) $\frac{(n-1)x^2 + x}{n}$

6.2 (1) (a)(b)(c) 略 (2)(a) $(-1)^n \frac{{}_{2n}C_n}{2}$ (b) $(-1)^n {}_{2n}C_n$

6.3 1080

6.4 66

第2章 解析幾何

2.1 平面図形

1.1 略

$$1.2 \quad (1)(x-2)^2 + (y-4)^2 = 4 \quad (2)y = x + 6 \pm 2\sqrt{2} \quad (3)2\sqrt{5} + 2 \quad \left(\frac{10 + 2\sqrt{5}}{5}, \frac{20 + 4\sqrt{5}}{5} \right)$$

$$1.3 \quad a = -1, -4, \quad b = 0$$

$$1.4 \quad \text{図略} \quad 0 < a \leq b \text{ のとき, } 4\pi b^2 \quad a < b \leq \sqrt{2}a \text{ のとき, } 4\pi b^2 - 4b^2 \cos^{-1} \frac{a}{b} \\ \sqrt{2}a < b \text{ のとき, } 4a^2 + \pi b^2 + 4a\sqrt{b^2 - a^2} + 4b^2 \sin^{-1} \frac{a}{b}$$

$$1.5 \quad |a| < 1$$

$$1.6 \quad (y-1)^2 = -4x + 1, \quad \text{グラフ: 略}$$

$$1.7 \quad (a, b), \quad a = -\frac{st + tu + us}{2}, \quad b = \frac{s + t + u}{2}$$

$$1.8 \quad \lambda = \mu = -2, \quad 2x + y - 2 = 0, \quad x - 2y + 1 = 0$$

$$1.9 \quad x + 3y = 0$$

2.2 直線・平面

$$2.1 \quad (\text{イ}) \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \quad (\text{ロ}) (1, 2, 3)/7$$

$$2.2 \quad 2x + y + z = 5$$

$$2.3 \quad 9x - 3y + 5z = 35$$

2.4 略

$$2.5 \quad (3, -2, -1)$$

$$2.6 \quad \frac{\alpha\beta\gamma}{6}$$

$$2.7 \quad \sqrt{14}$$

$$2.8 \quad (1) \left(\frac{1}{2}, 1, \frac{3}{2} \right) \quad (2) \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \quad (3) (0, 1, 2) \quad (4) \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right)$$

$$2.9 \quad \left(0, \frac{b - ay}{x}, \frac{c - az}{x} \right)$$

$$2.10 \quad x + y + z = 1, \quad \text{図略}, \quad \frac{11\sqrt{3}}{3}$$

$$2.11 \quad x + y - z = x_0 + y_0 - z_0$$

2.3 球

3.1 (1) $2x + y + 2z - 10 = 0$ (2) $\frac{|2a + b + 2c - 10|}{3}$ (3) 5

3.2 $x + 2y + 3z = 14$

2.4 空間曲線

4.1 略

2.5 2次曲面

5.1 9π

5.2 (1) $Ax_1x + By_1y + Cz_1z = 1$

(2) $\mathbf{n} = (Ax_1, By_1, Cz_1) / \sqrt{A^2x_1^2 + B^2y_1^2 + C^2z_1^2}, \quad \frac{1}{\sqrt{A^2x_1^2 + B^2y_1^2 + C^2z_1^2}}$

5.3 (1) $(4 - a^2)x^2 + 2ax + y^2 = 1$ (2) $\pm\sqrt{3}$ (3) $\pm\sqrt{\frac{3}{2}}$

5.4 $y = \frac{x^2}{16} - 1$

第3章 線形代数

3.1 ベクトル

縦ベクトルを ${}^t(1, 2, 3)$ のように表す

$$1.1 \quad t = -\frac{a \cdot b}{|b|^2} \text{ のとき、最小値 } \frac{||b|^2 a - (a \cdot b)b|}{|b|^2}$$

$$1.2 \quad p = -x - 2y - z, \quad q = 2x + 3y + 4z, \quad r = x + y + 2z$$

$$1.3 \quad (1)y = 3(x - x_0) + y_0 \quad (2)5k + 4t + 18 = 0 \quad (3)(4t - k)^2 + (2k + 6)^2 = 25 \\ (4)k = \frac{-3 \pm \sqrt{10}}{4}, \quad t = -\frac{3}{4} \mp \frac{5\sqrt{10}}{16}$$

1.4 略

$$1.5 \quad -\frac{3}{4}$$

3.2 一次結合

2.1 略

$$2.2 \quad |x \ y \ z| = 0$$

$$2.3 \quad (1)a \neq 0, \ b \neq 0 \quad (2)a = \mp \frac{1}{2}, \ b = -c = \pm \frac{1}{2}, \text{ 行列 (4 つ) は略}$$

$$2.4 \quad 1, -\frac{1}{2}$$

$$2.5 \quad \lambda \neq 1, -\frac{1}{2}$$

2.6 1 次独立

2.7 略

2.8 一次従属

2.9 一次従属

$$2.10 \quad 1, -2$$

2.11 一次独立

$$2.12 \quad (1)x = 2 \quad (2) \quad (3) \text{ 略}$$

$$2.13 \quad a = 1, \ x = 2$$

3.3 ベクトルの応用

3.1 前半略, $a = 1, b \pm 1, c = -2$

3.2 (1) 3 (2) $\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$ (3) $\cos^{-1}\left(\frac{10}{3\sqrt{21}}\right)$ (4) $\mathbf{j} - 3\mathbf{k}$

3.3 (1) $\frac{x_1x_2 + y_1y_2}{\sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}}$ (2) $\frac{|x_1y_2 - x_2y_1|}{2}$ (3) $\frac{|x_1y_2 + x_1y_3 + x_3y_1 - x_1y_3 - x_2y_1 - x_3y_2|}{2}$

3.4 (1) $a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0) = 0$ (2) $\frac{(a\mathbf{i} + b\mathbf{j} + c\mathbf{k})}{\sqrt{a^2 + b^2 + c^2}}$ (3) $\frac{a(x_1 - x) + b(y_1 - y) + c(z_1 - z)}{\sqrt{a^2 + b^2 + c^2}}$
 (4) $x + y + 2z - 13 = 0, \frac{13}{2}, (13, 0, 0), (0, 13, 0), (0, 0, \frac{13}{2})$

3.5 (1) $\frac{(|\vec{a}| |\vec{b}| + |\vec{b}| |\vec{a}|)}{||\vec{a}| |\vec{b}| + |\vec{b}| |\vec{a}|}$ (2) $\frac{x(\vec{b} - \vec{a})}{|\vec{b} - \vec{a}|}$ (3) $\frac{|\vec{a}| |\vec{b}| + |\vec{b}| |\vec{a}|}{|\vec{a} + \vec{b}|}$

3.6 (1) $\overrightarrow{BC} = \vec{b} - \vec{a}, \overrightarrow{BL} = (m - 1)\vec{a}, \overrightarrow{CM} = (n - 1)\vec{b}, \overrightarrow{BM} = n\vec{b} - \vec{a}, \overrightarrow{CL} = m\vec{a} - \vec{b}$
 (2) $\overrightarrow{BN} = s(n\vec{b} - \vec{a}), \overrightarrow{CN} = t(m\vec{a} - \vec{b})$ (3) $s = \frac{m - 1}{mn - 1}, t = \frac{n - 1}{mn - 1}$

3.7 略

3.8 (1) $(\overrightarrow{OB} - \overrightarrow{OA}) \perp \overrightarrow{OP}$ (2) 略

3.4 空間の基

4.1 $B = {}^tA$

4.2 (1) ${}^t(1, -1, 0, 1), {}^t(1, 0, -1, 1)$ (2) $\frac{1}{\sqrt{3}} {}^t(1, 1, 1, 0), \frac{1}{\sqrt{15}} {}^t(2, -1, -1, -3)$

4.3 $3 {}^t(1, 2, 3) - 5 {}^t(4, 3, 2) - 5 {}^t(3, 4, 6)$

4.4 (1) $1, {}^t(5, 1, 16, -14)$ (2) 3

4.5 (1) 3 (2) ${}^t(1, 1, 1, 0, 0), {}^t(0, 0, 0, 1, 1)$

3.5 行列

5.1 (1) $\begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 7 \end{pmatrix}$ (3) $\begin{pmatrix} 32 & 72 & 74 \\ 20 & 44 & 46 \\ 14 & 30 & 32 \end{pmatrix}$

5.2 $\begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$

5.3 $AB=BA$ なら AB も対称行列、 $AB \neq BA$ なら AB は対称行列にならない。

5.4 (1) $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$ (2) $f = 2u^2 + (1 + \sqrt{2})v^2 + (1 - \sqrt{2})w^2$

5.5 略

$$5.6 \begin{pmatrix} 0 & 3 & 2 & 0 \\ -3 & -4 & -4 & -1 \\ -1 & 0 & -2 & 1 \\ 2 & 1 & 2 & -2 \end{pmatrix}$$

$$5.7 \begin{pmatrix} 10 & 8 & 4 \\ 8 & 10 & 4 \\ 4 & 4 & 10 \end{pmatrix} \quad \begin{pmatrix} 10 & 8 & 4 \\ 8 & 10 & 4 \\ 4 & 4 & 10 \end{pmatrix}$$

5.8 (1) $\forall f, g \in V, \alpha : \text{定数に対して、} Tf = \int_{-1}^1 (t-x)^2 f(t) dt \text{ であるから、} T(f+g) = Tf + Tg, T(\alpha f) = \alpha Tf \text{ は明らかに成立する。}$

$$(2) T = \begin{pmatrix} \frac{2}{3} & 0 & 2 \\ 0 & -\frac{4}{3} & 0 \\ \frac{2}{5} & 0 & \frac{2}{3} \end{pmatrix}$$

$$5.9 (1) 5 \quad (2) T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos a & \sin a & 0 & 0 \\ 0 & -\sin a & \cos a & 0 & 0 \\ 0 & 0 & 0 & \cos 2a & \sin 2a \\ 0 & 0 & 0 & -\sin 2a & \cos 2a \end{pmatrix}$$

$$5.10 A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 4 & 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 8 & -2 & -1 \end{pmatrix}$$

$$5.11 \begin{pmatrix} 1 & -2 & -3 \\ -2a & 2a & 4a \\ ab & -ab & -2ab \end{pmatrix}$$

3.6 行列のn乗

6.1 $|a-b| < 1, |a+2b| < 1$ なら、零行列 O

$$|a-b| < 1, a+2b=1 \text{ なら、} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$a-b=1, |a+2b| < 1 \text{ なら、} \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}$$

$a-b=1, a+2b=1$ すなわち $a=1, b=0$ なら、単位行列 E
その他では、発散

$$6.2 (1) \text{ 略} \quad (2) \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(3) \begin{pmatrix} \lambda^n & a & b \\ 0 & \lambda^n & a \\ 0 & 0 & \lambda^n \end{pmatrix} \quad a = n\lambda^{n-1}, b = \frac{n(n-1)}{2}\lambda^{n-2}$$

$$6.3 \quad \frac{1}{1+t^2} \begin{pmatrix} 1 & t \\ -t & 0 \end{pmatrix}$$

$$6.4 \quad (1) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2) \text{ 略}$$

$$6.5 \quad (1) A^n = \begin{pmatrix} 1 & a & b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \quad a = {}_nC_1, \quad b = {}_{n+1}C_2 \quad (2) \text{ 略}$$

$$6.6 \quad A^2 = \begin{pmatrix} 1 & 2a \\ 0 & 1 \end{pmatrix} \quad A^3 = \begin{pmatrix} 1 & 3a \\ 0 & 1 \end{pmatrix} \quad A^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$$

$$6.7 \quad A^{50} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$6.8 \quad (a) \frac{1}{\alpha} \begin{pmatrix} 1 & \alpha-1 \\ 0 & \alpha \end{pmatrix} \quad (b) 0 \quad (c) \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

$$6.9 \quad (1) \alpha = \beta - 1 \quad (2) \beta = 2 \quad (3) \begin{pmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{pmatrix}$$

$$6.10 \quad (1) A^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad A^n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (n \geq 3)$$

$$(2) \begin{pmatrix} 1 & n & a \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix} \quad a = \frac{n(n-1)}{2}$$

6.11 略

3.7 行列と図形

$$7.1 \quad \tan 2\theta = -\frac{2b}{a-c}$$

$$7.2 \quad \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

7.3 (1) 略 (2) 原点を中心に角度 θ だけの回転

7.4 略

7.5 (b) 原点を中心に角度 60° だけの回転 (d) x 軸対称

$$7.6 \quad (1) \cos \theta' = \frac{2(a+b)}{\sqrt{(4a^2+1)(4b^2+1)}} \quad (2) \text{ 略} \quad (3) a = \pm \frac{\sqrt{3}}{2}, \quad b = \mp \frac{\sqrt{3}}{2}$$

7.7 (1) 略 (2) $T(2, -2, 1)$

3.8 逆行列

8.1 (1) $\frac{1}{2} \begin{pmatrix} -1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ (2) $\frac{1}{13} \begin{pmatrix} 4 & 8 & -1 \\ -2 & -17 & 7 \\ -3 & -6 & 4 \end{pmatrix}$

(3) $a \neq -1$ のとき、正則 $\frac{1}{a+1} \begin{pmatrix} a+1 & 1 & -2a-1 \\ 0 & 1 & 1 \\ 0 & -1 & a \end{pmatrix}$ (4) $\frac{1}{3} \begin{pmatrix} 3 & -9 & 3 \\ 3 & -1 & -2 \\ -3 & 8 & -2 \end{pmatrix}$

8.2 (1) $\frac{1}{8} \begin{pmatrix} 0 & 4 & 4 \\ 4 & -5 & -3 \\ 4 & -3 & -5 \end{pmatrix}$ (2) $\frac{1}{27} \begin{pmatrix} 3 & 6 & 6 \\ 6 & -6 & 3 \\ 6 & 3 & -6 \end{pmatrix}$ (3) $\frac{1}{2} \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$

(4) $\frac{1}{258} \begin{pmatrix} -20 & 35 & -1 \\ 8 & -14 & 52 \\ 34 & 5 & -37 \end{pmatrix}$ (5) $\frac{1}{16} \begin{pmatrix} -4 & 4 & 4 \\ 4 & -8 & 4 \\ 4 & 4 & -4 \end{pmatrix}$ (6) $\frac{1}{4} \begin{pmatrix} 4 & 2 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

(7) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ (8) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$ (9) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 4 \end{pmatrix}$

8.3 $\alpha \neq 0$

8.4 a, b, c, d のうちの 2 つも等しくない。

8.5 (1) 0 (2) $ad - bc \neq 0$ (3) $\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

8.6 略

8.7 (1) $\begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$ (2) $\begin{pmatrix} n & b \\ 0 & n \end{pmatrix} b = \frac{n(n+1)a}{2}$ (3) $\frac{1}{n^2} \begin{pmatrix} n & -b \\ 0 & n \end{pmatrix}$

8.8 略

8.9 $(a-b)(b-c)(c-a) \neq 0$ $\frac{1}{(a-b)(b-c)(c-a)} \begin{pmatrix} bc(c-b) & ca(a-c) & ab(b-a) \\ b^2 - c^2 & c^2 - a^2 & a^2 - b^2 \\ c-b & a-c & b-a \end{pmatrix}$

8.10 略

8.11 $\frac{1}{7} \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix}$

3.9 行列式

9.1 (1) 0 (2) $(4a+2b+c)(2b+c)c^2$ (3) $(x-y)(y-z)(z-x)$ (4) 9 (5) 5 (6) $(a^2+be)cd$

(7) 71 (8) $a_1^2 b_1^2 + a_2^2 b_2^2 + a_3^2 b_3^2 - 2(a_1 a_2 b_1 b_2 + a_2 a_3 b_2 b_3 - a_3 a_1 b_3 b_1)$ (9) 2 (10) $-\frac{1}{432}$

(11) $(x+a+b+c)(x-a)(x-b)(x-c)$ (12) 1400 (13) $1+a^2+b^2+c^2+d^2$ (14) $x^3(x-10)$

9.2 (1) 0 (2) 0

9.3 (1) $-2(a+b)(b+c)(c+a)$ (2) $(a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$
 (3) $(x-y)(y-z)(z-x)\{1-(xy+yz+zx)+xyz(x+y+z)-x^2y^2z^2\}$
 (4) $x(x+1)(x-1)^2$ (5) $(x-2)^2(x-3)$ (6) $(1-x)(1-y)(1-z)(x-y)(y-z)(z-x)$
 (7) $-(a-b)(x-a)(x-b)(x-a-b)$ (8) $4abc$ (9) $2(\cos\alpha-\cos\beta)(\cos\beta-\cos\gamma)(\cos\gamma-\cos\alpha)$
 (10) $(a+b+c)(c-a-b)(a-b-c)(b-c-a)$ (11) $(2x+1)(2x-1)$

9.4 (1) $a=b$ のとき、不定 $a \neq b$ のとき、 $x=a, b$ (2) $2a+1, 1-a$

9.5 (1) $a^{n-1}(a+nb)$ (2) $n!$

9.6 (1) 1 (2) 略

9.7 略

9.8 $g=xyzf, h=(x+y+z)f$

9.9 (1) $(a+x)^6 - a^6 = 0$ (2) $x=0, a\omega^k - a \quad k=1 \sim 5, \omega \neq 1$ (3) $y = \frac{1}{\omega^k - 1}$

9.10 $b+c$

9.11 (1) $4|A|, 2(|A| + |A|^{-1})$ (2) 4

9.12 1, -1

9.13 (1) $\begin{pmatrix} -3 & 2 \\ -1 & 3 \end{pmatrix}$ (2) $\pm\sqrt{2}$

9.14 -9

3.10 行列式の応用

10.1 略 $A = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}, B = \begin{pmatrix} x_1^2 + y_1^2 & y_1 & 1 \\ x_2^2 + y_2^2 & y_2 & 1 \\ x_3^2 + y_3^2 & y_3 & 1 \end{pmatrix}, C = \begin{pmatrix} x_1^2 + y_1^2 & x_1 & 1 \\ x_2^2 + y_2^2 & x_2 & 1 \\ x_3^2 + y_3^2 & x_3 & 1 \end{pmatrix}, D =$
 $\begin{pmatrix} x_1^2 + y_1^2 & x_1 & y_1 \\ x_2^2 + y_2^2 & x_2 & y_2 \\ x_3^2 + y_3^2 & x_3 & y_3 \end{pmatrix}$ とおく。
 中心 $\left(\frac{B}{2A}, -\frac{C}{2A}\right)$ 半径 $\frac{(B^2 + C^2 + 4AD)}{4A^2}$

10.2 (1) 略 (2) 3点 $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, 放物線

10.3 略

10.4 略

10.5 略

10.6 0

3.11 階数と方程式

- 11.1 (1) 不能 (2) $m = 0$ なら、 $x_1 + x_2 + x_3 = 0$, $m = -3$ なら、不能、その他なら、
 $x_1 = \frac{m-2}{m+3}$, $x_2 = x_3 = -\frac{5}{m+3}$ (3) $x = k - 22$, $y = k - 8$, $z = -k$, $u = 8$ ($\forall k \in \mathbf{R}$)
 (6) $a = 1$ なら、 $x + y + z = 1$, $a = -2$ なら、不能、その他なら、 $x = -\frac{a+1}{a+2}$,
 $y = \frac{1}{a+2}$, $z = \frac{(a+1)^2}{a+2}$ (7) $x_1 = 3$, $x_2 = \frac{1}{2}$, $x_3 = -3$ (8) $x_1 = k$, $x_2 = 1 - 2k$,
 $x_3 = 3k - 1$, $x_4 = -k + 2$ ($\forall \in \mathbf{R}$) (9) $x_1 = 1$, $x_2 = 0$, $x_3 = -2$

- 11.2 (1) $k = -3$ (2) 1 次元

- 11.3 $m = 2$, $-\frac{16}{5}$

- 11.4 略

- 11.5 (1) $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$, $Dx_1 = \begin{vmatrix} a_1 & a_{12} \\ a_2 & a_{22} \end{vmatrix}$, $Dx_2 = \begin{vmatrix} a_{11} & a_1 \\ a_{21} & a_2 \end{vmatrix}$ (2) $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{a_1}{a_2}$

- 11.6 略

- 11.7 (1) $|M| = (a + b + c)(c - a - b)(a - b - c)(b - c - a)$ (2) $MX = \begin{pmatrix} ax_2 + bx_3 + cx_4 \\ ax_1 + cx_3 + bx_4 \\ bx_1 + cx_2 + ax_4 \\ cx_1 + bx_2 + ax_3 \end{pmatrix}$
 (3) $|M| x_1 = -a(b - c)^2$, $|M| x_2 = a(c - b)(a - b)$, $|M| x_3 = -(b + c)(b - c - a)(c - a - b)$,
 $|M| x_4 = (b + c)(b - c - a)(c - a - b)$,
 (4) $|M| = 0$

- 11.8 3

- 11.9 $x = 2$, $y = -3$, $z = 1$

- 11.10 (1) $\frac{1}{30} {}^t(17, 0, -5)$ (2) 2

3.12 固有値と固有ベクトル

- 12.1 $-1, -2, 4$

- 12.2 $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ $2, \frac{1 \pm \sqrt{3}i}{2}$

- 12.3 $2, -1$

- 12.4 略

- 12.5 (1) $a \neq 1$ のとき、 $R(A) = 3$, $a = 1$ のとき、 $R(A) = 2$ (2) $a \neq 1$ のとき、 $x = 0$,
 $a = 1$ のとき、 $x = {}^t(-1, -1, 1)$

- 12.6 略

- 12.7 略

- 12.8 (1) 略 (2) 略

$$12.9 \quad (1) \begin{pmatrix} 19-\lambda & 22 \\ 43 & 50-\lambda \end{pmatrix} \quad (2) \lambda = \frac{69 \pm \sqrt{4745}}{2}$$

$$12.10 \quad (1) B = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix} \quad B' = \frac{1}{3} \begin{pmatrix} -1 & -3 & 3 \\ -3 & -1 & 3 \\ 3 & 3 & 2 \end{pmatrix}$$

$$\lambda = 1, \pm\sqrt{3} \quad J = \frac{10}{3}$$

$$(2) -(\lambda'_1 \lambda'_2 + \lambda'_2 \lambda'_3 + \lambda'_3 \lambda'_1)$$

$$(3) -\{(\lambda_1 - b_0)(\lambda_2 - b_0) + (\lambda_2 - b_0)(\lambda_3 - b_0) + (\lambda_3 - b_0)(\lambda_1 - b_0)\}$$

$$12.11 \quad (1) t = \frac{1}{2} \text{ のとき、階数 } 1 \quad t \neq \frac{1}{2} \text{ のとき、階数 } 2$$

$$(2) \tau = t_1 + t_2 - 2t_1 t_2 \quad (3) 1 \text{ と } 2t - 1$$

12.12 略

3.13 固有ベクトル

固有値の順に固有ベクトルを並べる

$$13.1 \quad (1) 1, 2, 5 \quad {}^t(1, 0, 1), {}^t(1, 1, -1), {}^t(1, -2, -1) \quad (2) 1, 4 \quad {}^t(1, 1, 1), {}^t(1, -1, 0), {}^t(1, 0, -1)$$

$$(3) 1, 3 \quad {}^t(1, -1), {}^t(1, 1) \quad (4) 3 \quad {}^t(1, 2)$$

$$(5) 1, -1 \quad {}^t(1, 1), {}^t(1, -1) \quad (6) a, a \pm \sqrt{2} \quad {}^t(1, 0, 1), {}^t(1, \sqrt{2}, 1), {}^t(1, -\sqrt{2}, 1)$$

$$(7) 0, 1 \quad {}^t(1, -1, 1), {}^t(1, -1, 0), {}^t(0, 1, -1) \quad (8) 0, 1, -2 \quad {}^t(1, 0, 0), {}^t(1, 1, 1), {}^t(1, -2, 4)$$

$$(9) 3, -1 \quad {}^t(1, 1), {}^t(1, -1)$$

$$13.2 \quad 3, 2, 1 \quad {}^t(1, 0, 0), {}^t(0, 1, 0), {}^t(1, -1, -1)$$

$$13.3 \quad (1) (x-2)(x-3)^2 \quad (2) \lambda = 2, 3 \quad {}^t(1, 1, 1), {}^t(1, 2, 1), {}^t(1, 0, -1)$$

$$13.4 \quad 1, 2, 3 \quad {}^t(7, -5, -11)$$

13.5 略

13.6 略

$$13.7 \quad (1) 8 \quad (2) \frac{1}{8} \begin{pmatrix} 4 & -2 & -2 \\ -2 & 5 & 1 \\ -2 & 1 & 5 \end{pmatrix} \quad (3) 1, 2, 4 \quad {}^t(1, -1, -1), {}^t(0, 1, -1), {}^t(2, 1, 1)$$

$$13.8 \quad 1, \frac{-1 \pm \sqrt{3}i}{2}, {}^t(1, -1, 1)$$

$$13.9 \quad (1) \text{略} \quad (2) \begin{pmatrix} I & 0 \\ A & I \end{pmatrix} \quad (3) 1, 1, -1, 8, \quad {}^t(2, 5, 0, -2), {}^t(6, 1, -2, 0), {}^t(0, 0, 1, -1), {}^t(0, 0, 2, 1)$$

$$13.10 \quad (1) \text{略} \quad (2) x = |\mathbf{a}|^2 \quad (3) 1, n+1(n-1 \text{ 重解}) \quad (1, 1, 1, 1, \dots, 1),$$

$$x_i = 1, x_{i+1} = -1, x_j = 0, (j \neq i, i+1, i = 1 \sim n-1)$$

3.14 固有空間

対角線以外の成分がすべて 0 であるとき、 $\text{diag}(1, 2, 3)$ のように表す

14.1 $c - ae = d = 0$

14.2 (1) 1, -2 (2) 平面 $x + y + z = 0$, 直線 $x = y = z$ 図は略

14.3 (1) $V = \{(x, y, z) | x = y = z\}$ (2) $\frac{2\pi}{3}$

14.4 (1) 略 (2) $W \perp W'$

3.15 Jordan 標準形

15.1 (1) 2, 4 (2) 略 (3) $P = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

15.2 略

15.3 (1) 1, 7, (1, -1, 0), (1, 1, 2), (1, 1, -1) (2) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 2 & -1 \end{pmatrix}$

15.4 (1) A: 1, 3, ${}^t(1, -1)$, ${}^t(1, 1)$ B: $2 \pm \sqrt{7}$, ${}^t(1, -2 \pm \sqrt{7})$ (2) 略 (3) $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

15.5 (1) 2, 4 (2) ${}^t(1, 1)$, ${}^t(1, -1)$ (3) $\text{diag}(2, 4)$

15.6 $1 \pm \sqrt{2}i$, $\begin{pmatrix} -1 \\ 1 + \sqrt{2}i \end{pmatrix}$, $\begin{pmatrix} -1 \\ 1 - \sqrt{2}i \end{pmatrix}$, $\begin{pmatrix} 1 + \sqrt{2}i & 0 \\ 0 & 1 - \sqrt{2}i \end{pmatrix}$

15.7 (1) 0 (2) 0, $2 \pm i$, ${}^t(1, 1, 1)$, ${}^t(1, -1 \pm i, 1)$

15.8 略

15.9 (1) 4 (2) $\frac{1}{8} \begin{pmatrix} 8 & -4 & -4 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & -2 & 0 & 2 \end{pmatrix}$ (3) 1, 2, 4 (4) 可能 (5) $\text{diag}(1, 2, 1, 4)$

15.10 (1) 0, 1, 3, ${}^t(1, 0, -1)$, ${}^t(0, 1, 0)$, ${}^t(2, 3, 1)$ (2) $\text{diag}(0, 1, 3)$ (3) ${}^t(0, 1, 0)$

15.11 a) $\frac{1}{8} \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ b) 2, 4, ${}^t(1, 1)$, ${}^t(1, -1)$ c) $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$, $P^t A P = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$

15.12 (1) $\frac{1}{2a} \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (2) $a, -a, 2a$, (1, 0, -1), (1, -2, 1), (1, 1, 1)
(3) $P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ ${}^t P A P = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & 2a \end{pmatrix}$

15.13 $a < -\frac{1 + \sqrt{13}}{6}$, $\frac{\sqrt{13} - 1}{6} < a$

3.16 Jordan 標準形の応用

16.1 (1) $0, 2, 3$ (2) $\text{diag}(0, 2, 3)$

(3) $\frac{1}{6} \begin{pmatrix} -3 \cdot 2^n + 3 \cdot 3^n & 3 \cdot 2^n - 3^n & 3^n \\ 3 \cdot 2^n + 3 \cdot 3^n & -3 \cdot 2^n - 3^n & 3^n \\ 3 \cdot 3^n & -3^n & 3^n \end{pmatrix} \quad P = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & \sqrt{3} & \sqrt{2} \\ 1 & -\sqrt{3} & \sqrt{2} \\ -2 & 0 & \sqrt{2} \end{pmatrix}$

16.2 (1) $P = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{5} \\ 1/\sqrt{2} & 2/\sqrt{5} \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad (2) \text{略} \quad (3) = \begin{pmatrix} 2e - e^2 & -e + e^2 \\ 2e - 2e^2 & -e + 2e^2 \end{pmatrix}$

16.3 (1) $a = 1$ (2) $a > 1$ (3) $a > 1$

16.4 (1) -1 (重根), $2, (1, -2, 1), (1, 0, -1), (1, 1, 1)$ (2) $T = \begin{pmatrix} 1 & \sqrt{3} & \sqrt{2} \\ -2 & 0 & \sqrt{2} \\ 1 & -\sqrt{3} & \sqrt{2} \end{pmatrix}$

(3) $x^2 + y^2 - 2z^2 = 1$ (回転双曲面)

16.5 (1) $F = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & -3 & 1 \end{pmatrix}$

(2) $2, -2, 4, \quad e_1 = {}^t(1, 0, 0), \quad e_2 = {}^t(0, 1, -1)/\sqrt{2}, \quad e_3 = {}^t(0, 1, 1)/\sqrt{2}$

(3) $g = 2y_1^2 - 2y_2^2 + 4y_3^2$

16.6 (1) $\begin{pmatrix} 1 & -1 & -1 \\ 1 & -2 & -3 \\ 0 & 1 & 1 \end{pmatrix} \quad (2) \text{diag}(-1, 1, 2) \quad (3) \begin{pmatrix} -32 & 31 & 29 \\ -95 & 94 & 91 \\ 31 & -31 & -30 \end{pmatrix}$

3.17 総合問題

17.1 (1) $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} \quad (2) 2 \quad (3) 1, 2, \quad {}^t(1, -1)/\sqrt{2}, \quad {}^t(0, 1) \quad \text{図略} \quad (3) \begin{pmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{pmatrix}$

17.2 (1) $\begin{pmatrix} 40 & 26 & 23 & 64 & 42 \\ 42 & 46 & 48 & 82 & 44 \\ 10 & 10 & 14 & 20 & 8 \\ 31 & 19 & 11 & 59 & 41 \\ 30 & 13 & 6 & 58 & 46 \end{pmatrix} \quad (2) -450 \quad (3) 5 \quad (4) \text{略}$

17.3 (1) $1, \pm\sqrt{5}, \quad {}^t(-\sqrt{3}, 1, 0), \quad {}^t(1, \sqrt{3}, \sqrt{5}-1), \quad {}^t(1, \sqrt{3}, -\sqrt{5}-1) \quad (2) {}^t(1, 2, 2\sqrt{3})$

第4章 微分法

4.1 数列

1.1 (1) $\frac{3n^2 + 3n + 2}{2}$

1.2 (1) $\pm 2, \pm 2i$ (2) $-\frac{1}{2}, -\frac{\omega}{2}, -\frac{\omega^2}{2}$, ただし $\omega^3 = 1$ で $\omega \neq 1$ (3) 存在しない

1.3 $x \neq 1$ のとき、 $\frac{x\{1 - (n+1)x^n + nx^{n+1}\}}{(1-x)^2}$, $x = 1$ のとき、 $\frac{n(n+1)}{2}$

1.4 $[x]$ は x を超えない最大の整数を表す。 $n - \left[\frac{n-1}{3}\right], \quad n(n+1) - \left[\frac{n-1}{3}\right] \frac{2n-3-3\left[\frac{n-1}{3}\right]}{2}$

1.5 (1) $\lambda_1 = 4, \lambda = -6$ (2) 略 (3) $a_n = 2 \times 4^n + 3 \times (-6)^n, b_n = 4^n - 6 \times (-6)^n$

1.6 (1) 略 (2) 略 (3) 略 (4) 略

4.2 級数

2.1 (1) 1 (2) $\frac{1}{4}$

2.2 (1) $|x| < 1$ のとき、 $\frac{1}{x} \log(1-x)$ 、 $x = 0$ のとき、1、 $x = -1$ のとき、 $\log 2$ 、その他のとき、発散 (2) $\frac{3}{5}$ (3) 収束 (4) $\frac{2}{3}$

2.3 (1) 略 (2) 略 (3) 略

2.4 (1) 略 (2) a) 0, b) 0 (3) $x \neq 1$ のとき、a) $\frac{1-x^{n+1}}{1-x}$, b) $\frac{1-(n+1)x^n + nx^{n+1}}{(1-x)^2}$ 、 $x = 1$ のとき、a) n , b) $\frac{n(n+1)}{2}$ (4) $\frac{1}{1-x}, \frac{1}{(1-x)^2}$

2.5 略

2.6 $\frac{27}{4}$

2.7 2

2.8 4

4.3 漸化式

3.1 (1) 1 (2) $\frac{5 - \sqrt{21}}{2}$

3.2 3

$$3.3 \quad (1) a_{n+1} - a_n = \frac{a_n - a_{n-1}}{\sqrt{a_n + 1} + \sqrt{a_{n-1} + 1}} \quad (2) \text{略} \quad (3) \text{略} \quad (4) \frac{1 + \sqrt{5}}{2}$$

$$3.4 \quad (1) \sqrt{2} \quad (2) \text{略} \quad (3) \text{略}$$

$$3.5 \quad (1) x_n = A^{n-1} x_1 \quad (2) 3, 1, {}^t(3 \ 1), {}^t(1 \ 2) \quad (3) \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} \quad (4) \begin{pmatrix} 6 - 2^{1-n} & 3 \cdot 2^{1-n} - 3 \\ 2 - 2^{2-n} & 3 \cdot 2^{2-n} - 1 \end{pmatrix} \\ (5) 3, 1$$

4.4 極限值

$$4.1 \quad (1) 0 \quad (2) 0 \quad (3) 0 \quad (4) 0 \quad (5) 5/2 \quad (6) \sqrt{e} \quad (7) 0 \quad (8) e^{-2} \quad (9) -3/2$$

$$4.2 \quad (1) -1/2 \quad (2) 5 \quad (3) 1 \quad (4) 2 \quad (5) 2 \quad (6) 1/2 \quad (7) 5/2 \quad (8) 1/2 \quad (9) 2 \cos a \quad (10) -1/6$$

$$4.3 \quad (1) -1/2 \quad (2) 1 \quad (3) 1/2 \quad (4) \alpha > 1 \text{ のとき}, 0, \alpha = 1 \text{ のとき}, 1, \alpha < 1 \text{ で } \alpha \neq 0 \text{ のとき}, \\ \text{発散} \quad (5) \log a - \log b \quad (6) 2 \quad (7) e^a \quad (8) \log 3 \quad (9) -1/24 \quad (10) e^2 \quad (11) 2 \quad (12) 1/2$$

$$4.4 \quad (1) 1 \quad (2) e^{-2} \quad (3) 0 \quad (4) 1 \quad (5) 1$$

$$4.5 \quad (1) n \quad (2) 4 \quad (3) \sqrt{e}$$

$$4.6 \quad (1) 1/2 \quad (2) 1 \quad (3) e^{-1} \quad (4) 0 \quad (5) 1 \quad (6) -1 \quad (7) 1 \quad (8) 0 \quad (9) 0 \quad (10) 0 \quad (11) 0$$

$$4.7 \quad (1) 2$$

$$4.8 \quad |x| < 1 \text{ のとき}, 2x^2 - 1, x = 1 \text{ のとき}, 1, |x| > 1 \text{ のとき}, 1/x, x = -1 \text{ のとき}, \text{なし}$$

$$4.9 \quad 1$$

$$4.10 \quad (1) b_n = pb_{n-1} + (1-p)\lambda + q \quad (2) a_n = b_1 p^{n-1} + q/(p-1) \quad (3) |p| < 1 \text{ のとき}, 1/(p-1)$$

$$4.11 \quad (1) n(n+1)(2n+1)/6 \quad (2) a^2(n+1)(2n+1)/6n \quad (3) a^2/3$$

$$4.12 \quad (1) 1 \quad (2) \infty \quad (3) e$$

$$4.13 \quad \text{任意の } \varepsilon > 0 \text{ に対して } n_0 = n_0(\varepsilon) \text{ を定めて,}$$

$$n > n_0 \Rightarrow |a_n - \alpha| < \varepsilon$$

なるとき, α を数列 $\{a_n\}$ の極限という.

$$4.14 \quad (1) \text{略} \quad (2) \text{略} \quad (3) 0$$

$$4.15 \quad S_n = \frac{n+1}{2n}, T_n^2 = \frac{n^2-1}{12n^2}, \frac{1}{2}, \frac{1}{2\sqrt{3}}$$

$$4.16 \quad (1) \text{略} \quad (2) \text{略} \quad (3) \text{略}$$

4.5 連続性と微分可能性

- 5.1 (1) $x \neq 0$ では連続, 微分可能, $x = 0$ では連続であるが微分不可能
 (2) $x \neq 0$ では連続, 微分可能, $x = 0$ では h の値に関わらず連続であるが, $h = 1$ のときは微分不可能, $h > 1$ ならば微分可能

- 5.2 (1) $x \neq 0$ のとき, $f'(x) = 2x \sin(1/x) - \cos(1/x)$, $x = 0$ のとき, なし (2) $x = 0$ で不連続

- 5.3 (1) $\lim_{x \rightarrow a} f(x) = f(a)$ (2) $\lim_{\Delta x \rightarrow 0} \frac{f(a + \Delta x) - f(a)}{\Delta x} = f'(a)$

5.4 略

4.6 微分

- 6.1 (1) $\frac{2}{x^3} \exp\left(-\frac{1}{x^2}\right)$ (2) $\frac{1}{\sqrt{A+x^2}}$ (3) $\cos x - x \sin x$ (4) $\log(x^2+1) + \frac{2x^2}{x^2+1}$ (5) $a^x \frac{(a^2+x^2) \log a - x}{\sqrt{a^2+x^2}^3}$
 (6) $-\tan x$ (7) $\frac{2x+3}{\sqrt{x^2+3x+5}}$ (8) $e^{-x}(\cos x - \sin x)$ (9) $\frac{1}{a} \cos\left(\frac{x}{a}\right)$ (10) $\frac{1}{\sqrt{x^2+x-2}}$

- 6.2 (1) $\sqrt{a^2-x^2}$ (2) $\frac{5}{\sqrt{1-25x^2}}$ (3) $x > 0$ のとき, $-\frac{1}{\sqrt{1-x^2}}$, $x < 0$ のとき, $\frac{1}{\sqrt{1-x^2}}$ (4) 0
 (5) $\frac{a}{1+a^2x^2}$ (6) $\sin^{-1}\sqrt{1-x^2} - \frac{|x|}{\sqrt{1-x^2}}$ (7) $1+2x \tan^{-1} x$ (8) $\frac{e^x}{\sqrt{1-e^{2x}}}$ (9) $\frac{1}{2x\sqrt{x^2-1}}$
 (10) $\frac{1}{1+3\sin^2 \frac{x}{2}}$

- 6.3 (1) $x^x(1+\log x)$ (2) $x^{x-2}(1-\log x)$ (3) $\frac{1}{x\sqrt{x}} a^x \log a$

6.4 $y' = -\cot x$

- 6.5 (1) $y' = -2$, $y'' = 0$ (2) $y' = -\frac{(2x+y)y}{x^2+2xy}$, $y'' = \frac{12(x^2+xy+y^2)}{(x^2+2xy)^3}$

- 6.6 $F'(x) = \begin{vmatrix} f'_{11}(x) & f_{12}(x) & f_{13}(x) \\ f'_{21}(x) & f_{22}(x) & f_{23}(x) \\ f'_{31}(x) & f_{32}(x) & f_{33}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f'_{12}(x) & f_{13}(x) \\ f_{21}(x) & f'_{22}(x) & f_{23}(x) \\ f_{31}(x) & f'_{32}(x) & f_{33}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) & f'_{13}(x) \\ f_{21}(x) & f_{22}(x) & f'_{23}(x) \\ f_{31}(x) & f_{32}(x) & f'_{33}(x) \end{vmatrix}$

- 6.7 $x \neq 0$ のとき, $\frac{2}{x^3} e^{1/x^2}$, $x = 0$ のとき, 0

6.8 $\frac{1}{2}$

6.9 $y = a(c - \cos t)$

6.10 $g'(0) = 0$, $g''(0) = 0$

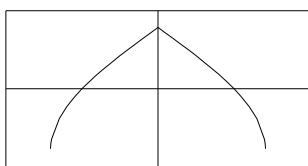
6.11 略

- 6.12 (i) $f' = \frac{1}{\sin x}$, $f'' = -\frac{\cos x}{\sin^2 x}$ (ii) $f' = \frac{1}{\sqrt{1-x^2}}$, $f'' = \frac{x}{(1-x^2)^{3/2}}$

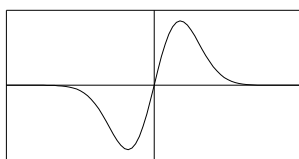
6.13 $4 \cos 2\theta \cos \theta - 2 \sin 2\theta \sin \theta$, $-\sqrt{2}$

4.7 グラフ

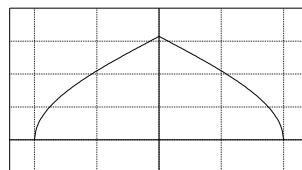
7.1



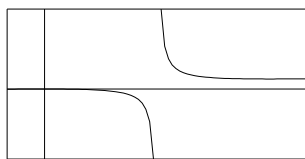
$$(1) y = \text{Sin}^{-1}(1 - x^2)$$



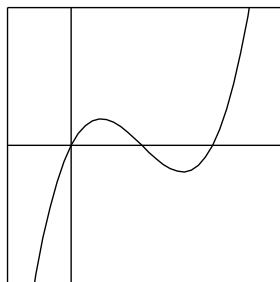
$$(2) y = x \exp(-x^2)$$



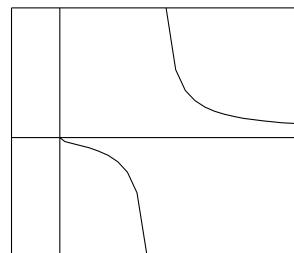
$$(3) y = \text{Sin}^{-1}\sqrt{1 - x^2}$$



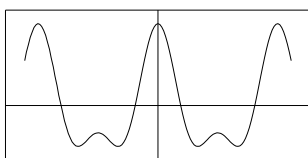
$$(4) y = \frac{x^2}{2(x - 3)}$$



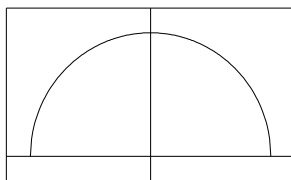
$$(5) y = x(x - 1)(x - 2)$$



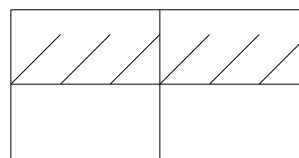
$$(6) y = \frac{1}{\log x}$$



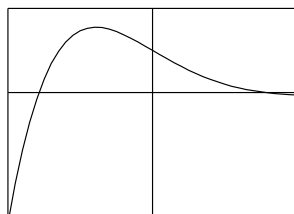
$$(7) y = 2 \cos x + \cos 2x$$



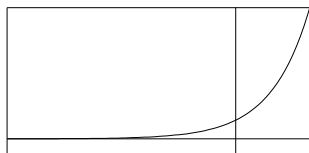
$$(8) y = \sqrt{1 - x^2}$$



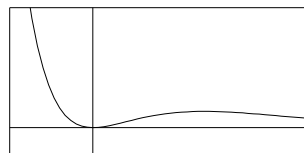
$$(9) y = x - [x]$$



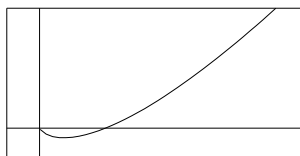
$$(10) y = e^{-x} \cos x$$



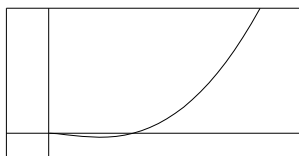
$$(11) y = \sinh x + \cosh x$$



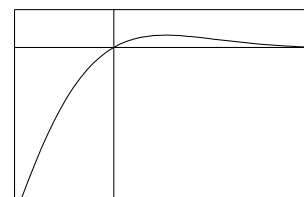
$$(12) y = x^2 e^{-x}$$



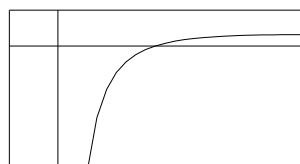
$$(13) y = x \log x$$



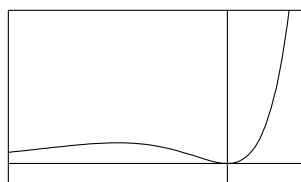
$$(14) y = x^2 \log x$$



$$(15) y = e^{-x} \sin x$$

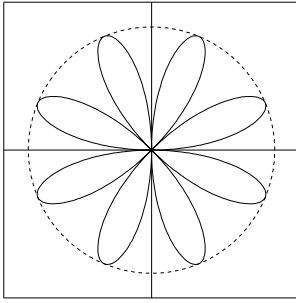


$$(16) y = \frac{\log x}{x}$$



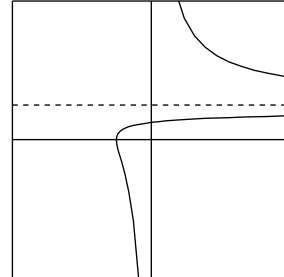
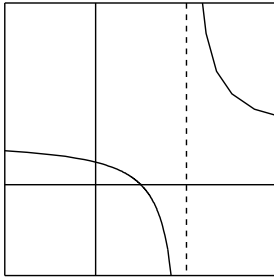
$$(17) y = x^2 e^x$$

7.2



$$r = |2 \sin \theta \cos \theta \cos 2\theta|$$

7.3



$$(1) x = 1 - 1/t, y = 1/(t+1)$$

$$(2) x = t^2 - 1, y = t/(t-1)$$

$$7.4 \quad (1) x = 2n\pi - \frac{\pi}{4} \text{ のとき極小値 } -\frac{e^{2n\pi - \pi/4}}{\sqrt{2}} \quad x = (2n+1)\pi - \frac{\pi}{4} \text{ のとき極大値 } \frac{e^{(2n+1)\pi - \pi/4}}{\sqrt{2}}$$

$$(2) x = 0 \text{ のとき, 極大値 } -1, \quad x = 2 \text{ のとき, 極小値 } 3$$

$$7.5 \quad y = 3x - 6 \pm 2\sqrt{10}$$

$$7.6 \quad x = 2n\pi - \frac{\pi}{4} \text{ のとき極小値 } -\frac{e^{2n\pi - \pi/4}}{\sqrt{2}} \quad x = (2n+1)\pi - \frac{\pi}{4} \text{ のとき極大値 } \frac{e^{(2n+1)\pi - \pi/4}}{\sqrt{2}}$$

7.7 略

$$7.8 \quad y' = \sqrt{t^2 + 1}, y'' = t\sqrt{t^2 + 1}, t > 0 \text{ なら下に凸}, t < 0 \text{ なら上に凸}$$

4.8 最大・最小

$$8.1 \quad x - y + z = 3$$

$$8.2 \quad (1) a \leq 0 \text{ なら } M = 0, m = a - 1, \quad 0 < a < 1 \text{ なら } M = \sqrt{\frac{a}{3^3}}, m = a - 1$$

$$1 \leq a < 3 \text{ なら } M = \sqrt{\frac{a}{3^3}}, m = 0, \quad 3 \leq a \text{ なら } M = a - 1, m = 0$$

$$8.3 \quad \text{半径 : 高さ} = 1 : \sqrt{2}$$

$$8.4 \quad \frac{a}{2}, \frac{\pi a^3}{3}$$

$$8.5 \quad (1) y' = -\frac{2x+y}{x+2y} \quad (2) -\frac{2(x^2-y^2)}{x+2y} \quad (3) x = y = \pm \frac{1}{\sqrt{3}} \text{ のとき, 最大値 } \frac{1}{3}, \quad x = -y = \pm 1$$

$$\text{のとき, 最小値 } -1$$

8.6 (1) $6 \sin x - 4 \sin^3 x$ (2) $x = \frac{\pi}{4}$ のとき, 最大値 $2\sqrt{2}$, $x = -\frac{\pi}{6}$ のとき, 最小値 $-\frac{5}{2}$

8.7 最大値 $\frac{2}{3\sqrt{3}}$, 最小値 $-\frac{2}{3\sqrt{3}}$

8.8 $\frac{2}{3}$ または $\frac{3}{2}$

8.9 (1) $\frac{\pi}{4} < a$ (2) $\sin a - \frac{2a}{\pi}$ (3) $\sqrt{1 - \frac{4}{\pi^2}}$

8.10 $x = 0$, $\frac{3}{2}$

8.11 $x = 3$, $y = \frac{3}{2}$, $\log 9 - \log 2$

4.9 微分の応用

9.1 略

9.2 $a^2 > 2$, $a^2 \geq 2b^2$

9.3 略

9.4 略

9.5 略

9.6 (1) 略 (2) 略

9.7 略

4.10 微分の応用 (力学)

10.1 略

10.2 $\frac{0.3}{a}$

4.11 n 次導関数

11.1 (1) $(x^2 + 20x + 90)e^x$ (2) $16e^x \sin x$

11.2 (1) $x^2 \cos \left(x + \frac{n\pi}{2}\right) - 2n \sin \left(x + \frac{(n-1)\pi}{2}\right) - n(n-1) \cos \left(x + \frac{(n-2)\pi}{2}\right)$ (2) $\sqrt{2^n} e^x \sin \left(x + \frac{n\pi}{4}\right)$
 (3) $\frac{(n-1)!}{x}$ (4) $x^3 \sin \left(x + \frac{n\pi}{2}\right) + 3nx^2 \sin \left(x + \frac{(n-1)\pi}{2}\right) + 3n(n-1)x \sin \left(x + \frac{(n-2)\pi}{2}\right) +$
 $n(n-1)(n-2) \sin \left(x + \frac{(n-3)\pi}{2}\right)$ (5) $\sum_{n=0}^{\infty} (2n)! 4^{-n} (n!)^{-2} x^n$ (6) $x \sin \left(x + \frac{n\pi}{2}\right) + n \sin \left(x + \frac{(n-1)\pi}{2}\right)$

11.3 (1) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ (2) $1 + \frac{x^2}{3} - \frac{x^4}{9} + \cdots$ (3) $x + \frac{x^2}{2} - \frac{2x^3}{3} + \cdots$ (4) $1 + \frac{x}{2} - \frac{x^2}{8} + \cdots$

11.4 (1) $f^{(2n)}(0) = 0$, $f^{(2n+1)}(0) = (-1)^n (2n)!$

$$11.5 \quad -x^3 + \frac{x^5}{2} - \cdots + (-1)^{n+1} \frac{x^{2n+3}}{(2n)!} + \cdots$$

11.6 略

$$11.7 \quad 1 + \binom{m}{1}x + \binom{m}{2}x^2 + \cdots + \binom{m}{n}x^n$$

$$11.8 \quad (1)y' = \frac{1}{\sqrt{1-x^2}} \quad (2)(1-x^2)y'' - xy' = 0 \quad (3) \text{ 略}$$

$$(4)y(0) = \frac{\pi}{2}, y^{(2)}(0) = y^{(4)}(0) = \cdots = y^{(2n)}(0) = 0 \quad y^{(2n-1)}(0) = (-1)^{n-1}((2n-1)!!)^2$$

$$11.9 \quad (1) \text{ 略} \quad (2)(1+x^2)y^{(n+2)} + n \cdot 2xy^{(n+1)} + n(n-1)y^{(n)} = 0$$

$$(3)y^{(2n)}(0) = 0, y^{(2n-1)}(0) = (-1)^{n-1}(2n-2)!$$

11.10 略

$$11.11 \quad (1)x - \frac{x^3}{3!} + \cdots \quad (2)a = 0, b = 1, c = 0, d = -\frac{1}{6}$$

11.12 略

11.13 略

$$11.14 \quad (1) \text{ 略} \quad (2)y' = \frac{1}{\sqrt{1-x^2}}, y'' = \frac{x}{\sqrt{(1-x^2)^3}} \quad (3) \text{ 略}$$

$$11.15 \quad (1)x^{2k} \text{ の係数 } {}_nC_k(-1)^{n-k} \quad (2)n:\text{奇数のとき} = 0, n:\text{偶数のとき} = (-1)^{n/2}n!2^{-n} \left(\left(\frac{n}{2}\right)!\right)^{-2}$$

$$(3) -\frac{n+1}{n+2}$$

$$11.16 \quad (1) \text{ 略} \quad (2) \text{ 略} \quad (3) \text{ 略}$$

4.12 近似式

$$12.1 \quad (1) \text{ 略} \quad (2) \text{ 略} \quad (3) \text{ 略}$$

$$12.2 \quad (1)1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} \cdots$$

$$(2)1 + x \log 4 + x^2 \frac{(\log 4)^2}{2} + x^3 \frac{(\log 4)^3}{3!} + \cdots$$

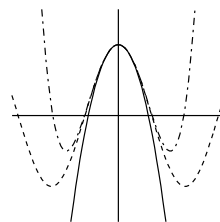
$$12.3 \quad 1 + \binom{m}{1}x + \binom{m}{2}x^2 + \cdots + \binom{m}{n}x^n$$

12.4 略

$$12.5 \quad \text{与式} = 1 + \sin \frac{x}{2} + \sin^2 \frac{x}{6} + \cdots, 1$$

12.6 略

$$12.7 \quad (1) \text{ 略} \quad (2) \text{ 右図} \quad (3) 0.00007$$



4.13 テイラー展開

13.1 (1) 略

$$13.2 (1) 1 + \frac{x^2}{2} + \frac{3x^4}{8} + \cdots + \binom{-1/2}{n} x^{2n} + \cdots \quad (-1 < x < 1)$$

$$(2) x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots + \binom{-1/2}{n} \frac{x^{2n+1}}{2n+1} + \cdots \quad (-1 < x < 1)$$

$$(3) x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots \quad (-1 < x \leq 1)$$

$$(4) x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} \cdots$$

$$13.3 (1) x - x^3 + x^5 - \cdots + (-1)^{2n-1} + \cdots \quad (-1 < x < 1)$$

$$(2) x + \binom{-1/2}{1} \frac{x^3}{3} + \binom{-1/2}{2} \frac{x^5}{5} + \cdots + \binom{-1/2}{n} \frac{x^{2n+1}}{2n+1} + \cdots \quad (-1 < x < 1)$$

$$(3) - \sum_{n=1}^{\infty} \frac{x^n}{n} \quad (-1 \leq x < 1)$$

$$(4) 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} + \cdots \quad (-1 < x < 1)$$

$$(5) \frac{1}{2\{2 \cdot 1 + 3 \cdot 2x + 4 \cdot 3x^2 + 5 \cdot 4x^3 + \cdots + (n+1)nx^{n-1} + \cdots\}} \quad (-1 < x < 1)$$

$$(6) x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots \quad (-1 < x \leq 1)$$

$$(7) 2x - \frac{17x^3}{2} + \cdots + (-1)^{n-1} \{3^{2n-1} + 2^{2n-1}(2n-1)\} \frac{x^{2n-1}}{(2n-1)!} + \cdots \quad (-\infty < x < \infty)$$

$$13.4 (1) n : \text{偶数のとき}, f^{(n)}(0) = 0, n : \text{奇数のとき}, (-1)^{(n-1)/2} n! \quad (2) \sum_{n=1}^{\infty} (-1)^{n-1} x^{2n-1}$$

$$13.5 f^{(n)} = (-1)^{n-1} \frac{(n-1)!}{(1+x)^n} \quad f = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots$$

$$13.6 \text{ 後半 : } \frac{\pi}{4} = 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$$

$$13.7 (1) \text{ 略 } (2) \text{ 与式 } \simeq t^{-2} + \frac{t^{-4}}{2} + \frac{t^5}{t^2-1} = 0.117$$

$$13.8 \text{ 前半略, 後半 } \sum_{n=0}^{\infty} 2^n \sin\left(\frac{n\pi}{6}\right) \frac{x^n}{n!}$$

$$13.9 1 - \frac{x}{2} - \frac{x^2}{4} + \cdots$$

$$13.10 x^5 \text{ の係数は } \frac{1}{5}, x^6 \text{ の係数は } -\frac{1}{6}$$

$$13.11 (1) \text{ 略 } (2) \text{ 略 } (3) f^{(2n-1)}(0) = (-1)^{n-1} ((n-1)!)^2 4^{n-1}$$

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n-1} ((n-1)!)^2 4^{n-1} \frac{x^{2n-1}}{(2n-1)!}$$

$$13.12 (1) \text{ 略 } (2) -1 < x < 1 \quad (3) 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} + \cdots \quad (-1 < x < 1)$$

$$13.13 f(x) = \sum_{n=0}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$$

$$13.14 \quad (1) \sum_{n=0}^{\infty} f^{(n)}(a) \frac{(x-a)^n}{n!} \quad (2) 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$(3) 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots$$

13.15 略

$$13.16 \quad x = \pm k, y = \pm \frac{1}{2}; \quad x = \pm \frac{1}{2}, y = \pm k \quad (\text{同順})$$

$$13.17 \quad (1) 1 + t + t^2 + t^3 + t^4 + \cdots + t^n + \cdots \quad (2) x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^{n-1} \frac{x^{n+1}}{n+1} + \cdots$$

4.14 総合問題

$$14.1 \quad (1) \pm \sqrt{a^2 m^2 + b^2} \quad (2) x^2 + y^2 = a^2 + b^2 \quad (3) 2a^2 + 2b^2$$

$$14.2 \quad \text{前半}, x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} = \frac{2x_{n-1}^3 + z}{3x_{n-1}^2}, \quad \text{後半略}$$

第5章 積分法

5.1 不定積分

1.1 (1) $x \log x - x + C$

1.2 (1) $\sqrt{x+1} + 2 \log |\sqrt{x+1} - 1| - \log |x|$ (2) $\frac{1}{2}\sqrt{6} \cdot \left\{ \log \left| \tan \frac{x}{2} - \sqrt{\frac{2}{3}} \right| - \log \left| \tan \frac{x}{2} + \sqrt{\frac{2}{3}} \right| \right\}$
 (3) $\frac{1}{2} \cdot \log |x^2 - 16|$ (4) $\frac{1}{8} \cdot \text{Sin}^{-1} x - \frac{x(1-2x^2)\sqrt{1-x^2}}{8}$ (5) $(x-1)e^x$ (6) $\frac{(\log x)^2}{2}$
 (7) $\frac{1}{2} \cdot \log \left| 2x + \frac{1}{4} + \sqrt{4x^2 + x + 1} \right|$ (8) $\frac{1}{1+e^x} + \log \frac{e^x}{1+e^x}$ (9) $-\frac{\sqrt{1-x^2}}{x} - \text{Sin}^{-1} x$
 (10) $\frac{(6x^2 - x + 1)\sqrt{1-x}}{15}$ (11) $\frac{\log x}{\log a}$ (12) $\text{Sin}^{-1} \frac{x}{a}$ (13) $\log(1+e^x)$

1.3 (1) $\tan^5 \frac{x}{5}$ (2) $\frac{x^2}{2} + x \tan x \log |\cos x|$ (3) $\frac{e^{3x}(3 \sin 2x - 2 \cos 2x)}{13}$ (4) $-\frac{e^{-x}(\sin x + \cos x)}{2}$
 (5) $x(\log x)^2 - 2x \log x + 2x^2$ (6) $\frac{e^{-x}(\sin x - \cos x)}{2}$ (7) $-\cot x$ (8) $\frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$
 (9) $2 \tan^{-1} \left(\frac{\tan \left(\frac{x}{2} \right)}{\sqrt{3}} \right)$ (10) $\log |\tan x|$ (11) $\frac{\log \left| \tan \left(\frac{x}{2} + \frac{\pi}{8} \right) \right|}{\sqrt{2}}$ (12) $\frac{-3x \cos 3x + \sin 3x}{9}$
 (13) $x^2 \sin x + 2x \cos x - 2 \sin x$ (14) $\frac{-2x \cos 2x + \sin 2x}{8}$

1.4 (1) $x \text{Sin}^{-1} x + \sqrt{1-x^2}$ (2) $x \tan^{-1} \log |1+x^2|$ (3) $x \sec^{-1} x + \log |x + \sqrt{x^2 - 1}|$

1.5 (1) $\log(1+\sqrt{1-x}) - \frac{1}{2} \log |x| - \frac{\sqrt{1-x}}{x}$ (2) $\frac{e^x}{1+x^2}$ (3) $2 \log |\sqrt{x+2} - \sqrt{x-3}| + 4 \tan^{-1} \left\{ \frac{1}{2} \sqrt{\frac{x+2}{x-3}} \right\}$
 (4) $2 \log |x+2 - \sqrt{x^2+x+1}| - \frac{1}{2} \log |2x+1 - 2\sqrt{x^2+x+1}| + \sqrt{x^2+x+1} - x$

1.6 (1) $3 \log |x+1| + \log(x^2+1) + \tan^{-1} x$ (2) $\frac{\log |x-2| - \log |x+2|}{4} - \tan^{-1} x$
 (3) $\frac{1}{2} \log |x(x+1)(x+2)|$ (4) $\frac{1}{2} \log \frac{(x-1)^2}{|x(x+2)|}$ (5) $-\frac{1}{4} \tan \left(\frac{x}{2} \right) + \log \frac{(x+1)^2}{x^2+4} - \frac{3}{x+1}$
 (6) $-\frac{1}{4} \tan^{-1} \frac{x}{2} + \log \frac{(x+1)^2}{x^2+4} - \frac{3}{x+1}$ (7) $\log \frac{x-2}{x-1}$ (8) $\frac{1}{2} \tan^{-1} x + \frac{1}{4} \log \left| \frac{x-1}{x+1} \right|$
 (9) $x + \frac{1}{2} \tan^{-1} x + \frac{1}{4} \log \left| \frac{x-1}{x+1} \right|$ (10) $\log |(x-2)^2(x+1)|$ (11) $\frac{x^2}{2} + 3x + \log \frac{(x-2)^{10}}{(x-1)^4}$

1.7 (1) $I_n = x(\log x)^n - I_{n-1}$, $I_4 = x(\log x)^4 - 4x(\log x)^3 + 12x(\log x)^2 - 24x \log x + 24$

1.8 (1) $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{t^2-1}{1+t^2}$, $dx = -\frac{2dt}{1+t^2}$ (2) $-\frac{\cot x}{2}$

1.9 (1) $I_n = \frac{1}{2} x^2 (\log x)^n - \frac{1}{2} n I_{n-1}$ (2) $I_n = \frac{1}{2n-1} \frac{\sin x}{\cos^{2n-1} x} + \frac{2n-2}{2n-1} I_{n-1}$

5.2 定積分

$$2.1 \quad (1) 2\sqrt{3} \quad (2) \frac{\sin 2t}{2} \quad (3) \frac{2\sqrt{3}\pi}{9} \quad (4) \frac{1}{2} \quad (5) \frac{\pi}{4} \quad (6) \log(1+x) \quad (7) \pi \quad (8) \frac{\pi}{4} \quad (9) \frac{4\sqrt{2}}{9} \\ (10) \sqrt[4]{8} - \sqrt[4]{2} + \tan^{-1} \sqrt[4]{2}$$

$$2.2 \quad (1) -\frac{1}{4} \quad (2) \pi \quad (3) 3 \log 3 - 2 \quad (4) \frac{4}{\pi^2} \quad (5) \sin t - t \cos t \quad (6) 2e - 2 \quad (7) 2 \log 2 - \frac{3}{4}$$

$$2.3 \quad (1) \frac{\sqrt{3}\pi}{9} \quad (2) \frac{3\pi}{40} - \frac{1}{20} \log \frac{9}{8}$$

$$2.4 \quad (1) \frac{\pi}{a} \tan^{-1} \frac{1}{a} \quad (2) \frac{\pi}{4}$$

$$2.5 \quad \frac{5}{12} + \frac{\pi + \sqrt{3}}{8}$$

$$2.6 \quad (1) 2xf(x^2) \quad (2) f(t, t) + \int_0^t f_t(t, x) dx \quad (3) (2x - k)f(kx^2 - k^2x) - kf(k^2x)$$

2.7 前半略 後半のすべての x の値に対して $f(x) = g(x)$ が成り立つとき

2.8 略

$$2.9 \quad n = 0 : \frac{(3\sqrt{3} - 2)\pi}{24\sqrt{3}}, \quad n = 1 : \frac{1}{4} \log \frac{3}{2}, \quad n = 2 : \frac{(2\sqrt{3} - 3)\pi}{24}, \quad 3 : \frac{1}{4} \log \frac{32}{27}$$

2.10 略

2.11 (1) 略 (2) 略

2.12 (1) 略 (2) 1

5.3 パラメータを含む定積分

$$3.1 \quad (1) \text{ 略} \quad (2) 1 \quad (3) -\frac{2n^2}{k^2(1 - e^{k/n})}$$

3.2 略

3.3 (1) 略 (2) $x = 0$ のときは値なし, $x \neq 0$ のとき 0 (3) 略

3.4 (1) 1 (2) 0

3.5 (1) $m = n$ のとき, π $m \neq n$ のとき, 0 (2) $m = n$ のとき, π $m \neq n$ のとき, 0 (3) 0

3.6 $n \neq m$ のとき, 0 $n = m$ のとき, $\frac{2}{2n+1}$

3.7 いずれでも 0

3.8 (1) $|\alpha| \geq 1$ のとき, $\alpha^2 - \frac{1}{3}$, $|\alpha| < 1$ のとき, $4\alpha^3 - \alpha^2 + \frac{1}{3}$ (2) $\alpha \pm \frac{1}{2}$

5.4 漸化式による定積分

$$4.1 \quad (1) n: \text{odd} \text{ のとき } , I_n = \frac{n-1}{n} \frac{n-3}{n-2} \cdots \frac{2}{3} \quad n: \text{even} \text{ のとき } , I_n = \frac{n-1}{n} \frac{n-3}{n-2} \cdots \frac{1}{2} \frac{\pi}{2}$$

$$4.2 \quad (1) k: \text{odd} \text{ のとき } , \frac{k-1}{k} \frac{k-3}{k-2} \cdots \frac{2}{3} \quad k: \text{even} \text{ のとき } , \frac{k-1}{k} \frac{k-3}{k-2} \cdots \frac{1}{2} \frac{\pi}{2} \quad (2) \pi$$

4.3 略

$$4.4 \quad (1) \text{ 略} \quad (2) I_2 = \frac{\pi}{4} \quad (3) I_{n+2} = \frac{n+1}{n+2} I_n \\ (4) n: \text{odd} \text{ のとき } , I_n = \frac{n-1}{n} \frac{n-3}{n-2} \cdots \frac{2}{3} \quad n: \text{even} \text{ のとき } , I_n = \frac{n-1}{n} \frac{n-3}{n-2} \cdots \frac{1}{2} \frac{\pi}{2}$$

$$4.5 \quad (1) 1 \quad (2) \text{ 略} \quad (3) 9e - 24$$

4.6 略

$$4.7 \quad \text{前半略} , \quad \text{後半 } n: \text{even} \text{ のとき } , I_n = \frac{1}{n-1} - \frac{1}{n-3} + \cdots + (-1)^{n/2-1} + (-1)^{n/2} \frac{\pi}{4} \\ n: \text{odd} \text{ のとき } , I_n = \frac{1}{n-1} - \frac{1}{n-3} + \cdots + \frac{(-1)^{(n-1)/2-1}}{2} (-1)^{(n-1)/2} \log \sqrt{2}$$

5.5 広義積分

$$5.1 \quad (1) \text{ 値なし} \quad (2) \frac{1}{3} \log 3 \quad (3) 2 \quad (4) \frac{2\sqrt{3}\pi}{3} \quad (5) \frac{1}{2} \log 2 \quad (6) \frac{\pi}{4} \quad (7) n! \quad (8) \frac{4\pi}{9\sqrt{3}} \quad (9) \frac{1}{a^2} \\ (10) 1 \quad (11) \frac{1}{4} \quad (12) \text{ 値なし}$$

$$5.2 \quad (1) \frac{1}{2} \quad (2) 2 \quad (3) 2\pi \quad (4) \pi$$

$$5.3 \quad (1) 3^{-2/3} \quad (2) 3$$

$$5.4 \quad (-1)^M \frac{M!}{(N+1)^{M+1}}$$

$$5.5 \quad \frac{1 + e^{-\pi}}{2(1 - e^{-\pi})}$$

$$5.6 \quad (1) \text{ 略} \quad (2) a^{-1} \quad (3) a^{-1}, a^{-2}$$

$$5.7 \quad (1) \frac{\pi}{2} \quad (2) 0 \quad (3) 0 \quad (4) \frac{\pi}{2}$$

$$5.8 \quad n: \text{even} \text{ なら } 0, \quad n: \text{odd} \text{ なら } \pi$$

5.9 略

5.6 特殊関数

6.1 略

$$6.2 \quad (1) \text{ 略} \quad (2) \frac{2^{-s}}{\Gamma(s)}$$

6.3 略

$$6.4 \quad (1) \text{ 略} \quad (2) 24$$

5.7 積分の応用 (最大・最小)

7.1 (1) $2\sqrt{c}$ (2) $2\pi a = h$

7.2 $x = 0, y = -2, I = \frac{4\pi(\pi - 3)}{3}$

7.3 $x = 0$ のとき, 最小値 0 , $x = \pm\sqrt{\pi}$ のとき, 最大値 $\frac{1 + e^{-\pi}}{2}$

7.4 (1) 右図 (2) $s < -1$ のとき, 0 , $-1 \leq s \leq 0$ のとき, $e^{-s}(e - e^{-s})$, $0 < s$ のとき, $e^{-s}(e - 1)$ (3) $0 = s$ のとき, 最大値は $e - 1$

7.5 $\frac{4\pi}{3\sqrt{3}}$

5.8 積分の応用 (図形)

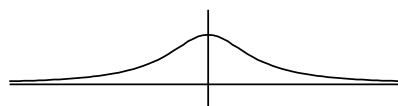
8.1 $a = \frac{8}{\pi^2}$, $b = 0$, $E = 1 - \frac{96}{\pi^4}$

8.2 (1) $\frac{(1 + e^{-\pi})e^{-k\pi}}{2}$ (2) $\frac{(1 + e^{-\pi})(1 - e^{-(n+1)\pi})}{2(1 - e^{-\pi})}$ (3) $\frac{1 + e^{-\pi}}{2(1 - e^{-\pi})}$ (4) $n = 1$

8.3 (1) 右図 (2) $a > 1$ なら, $\log \tanh a - \log \tanh 1$, $0 < a < 1$ なら, $\log \tanh 1 - \log \tanh a$, $a \leq 0$ なら, 0

8.4 $3\pi a^2$

8.5 (1) $x = 0$ のとき, 極大値 1 , $x = \pm\frac{1}{\sqrt{3}}, y = \frac{3}{4}$ のとき, 変曲点, 漸近線: $x = 0$, 右図
(2) $\frac{3\pi - 2}{6}$



8.6 (1) 右下図 (2) 略

8.7 $\frac{1}{6}$

8.8 (1) $S_n = \frac{2n(2n-1)}{4n^2+1} S_{n-1}$ (2) $S_n = \frac{(2n)!(1 - e^{-\pi})}{\prod_{k=1}^n (4k^2 + 1)}$

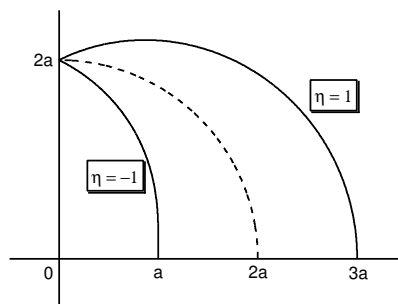
8.9 (1) $y' = -\frac{2x+y}{x+y}$, $y'' = -\frac{1}{(x+y)^3}$ (2) π

8.10 $\frac{3\pi}{2}$

8.11 πab

8.12 $\frac{3\pi a^2}{2}$

8.13 (1) $y = 3ex - 2e$ (2) $\frac{e}{3}$



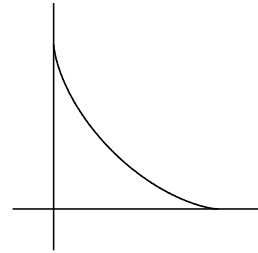
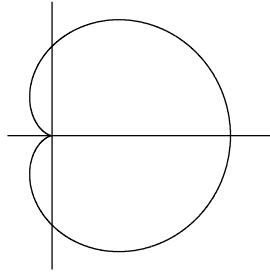
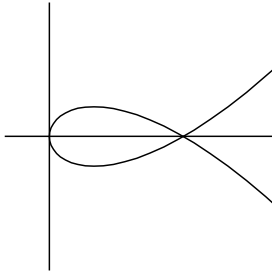
5.9 積分の応用 (長さ)

9.1 $6a$

9.2 (1) 下左図 (2) $\frac{8\sqrt{3}}{5}$

9.3 (1) 下中図 (2) $\frac{3\pi a^2}{2}$ (3) $8a$

9.4 (1) 下右図 (2) $\frac{3a}{2}$ (2) $\frac{3\pi a^2}{32}$



5.10 積分の応用 (回転体)

10.1 $5\pi^2 a^3$

10.2 $\frac{20\sqrt{2}\pi}{3}$

10.3 (1) 略 (2) $8a$ (3) $5\pi^2 a^3$

5.11 区分求積法

11.1 $\frac{\pi}{4}$

11.2 (1) 略 (2) 略

11.3 略

11.4 (1) $\frac{\pi}{4}$ (2) $\log 2$

11.5 (1) 略 (2) 0 (3) 略

5.12 積分の応用 (物理)

12.1 (1) $0 \leq t \leq \frac{\pi}{3}$ のとき, $H(t) = \sqrt[3]{\frac{3t}{\pi}}$, $\frac{\pi}{3} < t$ のとき, $H(t) = \frac{3\pi}{4\pi - 3t}$
(2) $0 \leq t \leq \frac{\pi}{3}$ のとき, $H'(t) = \frac{1}{\sqrt[3]{9\pi t^2}}$, $\frac{\pi}{3} < t$ のとき, $H'(t) = \frac{9\pi}{(4\pi - 3t)^2}$

5.13 積分の総合問題

13.1 (1) 図略 (2) 略 (3) $x\sin^{-1}x + \sqrt{1-x^2} + C$

13.2 (1) $\pm\sqrt{a^2m^2+b^2}$ (2) $x^2+y^2=a^2+b^2$ (3) $2(a^2-b^2)$

13.3 (1) 略 (2) 略 (3) 略

13.4 略

13.5 (1) $\vec{a} = a\vec{i} + \frac{\vec{j}}{\sqrt{2a+1}}$, $\vec{b} = 2a\vec{i} + \frac{\vec{j}}{\sqrt{4a+1}}$ $\vec{a} \cdot \vec{b} = 2a^2 + \frac{1}{\sqrt{8a^2+6a+1}}$
(2) $x + y\sqrt{2a+1}^3 = 3a+1$, $\frac{3a+1}{\sqrt{1+(2a+1)^3}}$
(3) 公式は略, $\frac{\pi}{2} \log \frac{4a+1}{2a+1} - \frac{\pi a}{3(2a+1)(4a+1)}$

第6章 偏微分法

6.1 偏微分

$$\begin{aligned}
 1.1 \quad (1) z_x &= 6x(x^2 + y)^2, z_y = 3(x^2 + y)^2 & (2) z_x &= \frac{\cos x}{\sin x + \cos y}, z_y = -\frac{\sin y}{\sin x + \cos y} \\
 (3) z_x &= -\frac{y}{x^2 + y^2}, z_y = \frac{x}{x^2 + y^2} & (4) z_x &= \frac{x^3}{(\sqrt{x^2 + y^2})^3}, z_y = \frac{y^3}{(\sqrt{x^2 + y^2})^3} \\
 (5) z_x &= -\frac{2xy}{(x^2 + y^2 + 1)^2}, \frac{x^2 - y^2 + 1}{(x^2 + y^2 + 1)^2} \\
 (6) z_x &= (\cos y - y \cos x) \cos(x \cos y - y \sin x), z_y = (-x \sin y - \sin x) \cos(x \cos y - y \sin x) \\
 (7) z_x &= -\frac{2y}{4x^2 + y^2}, z_y = \frac{2x}{4x^2 + y^2}
 \end{aligned}$$

$$1.2 \quad (1)0 \quad (2)0 \quad (3)0 \quad (4)0 \quad (5)4x^2 + 36xy + 4y^2 \quad (6)0 \quad (7)(a^2 - b^2)z$$

$$1.3 \quad (1)0$$

$$1.4 \quad \text{略}$$

$$1.5 \quad \text{略}$$

$$1.6 \quad \text{略}$$

$$1.7 \quad -\sin xy - xy \cos xy$$

$$1.8 \quad \text{略}$$

$$1.10 \quad (1) \frac{1 - 2xf'}{2zf' - 1} \quad (2) \frac{1 - 2yf'}{2zf' - 1} \quad (3) x - y$$

$$1.11 \quad z_{xx} = r^{-3}(x^2 r f'' + y^2 f'), z_{xy} = xyr^{-3}(r f'' - f'), z_{yy} = r^{-3}(y^2 r f'' + x^2 f')$$

$$1.12 \quad (1) z_x = \frac{1 - 2xf'}{2zf' - 1}, z_y = \frac{1 - 2yf'}{2zf' - 1} \quad (2) \text{略}$$

$$1.13 \quad f_x = 3x^2 a^{\sin y}, f_y = x^3 a^{\sin y} \log a \cdot \cos y, f_{xy} = 3x^2 a^{\sin y} \log a \cdot \cos y$$

$$1.14 \quad (1) u_x = \frac{3(x^2 - yz)}{x^3 + y^3 + z^3 - 3xyz} \quad (2) \text{略}$$

6.2 合成関数の偏微分

$$2.1 \quad (1) z_t = \frac{-y(1 + \cos t) + x \sin t}{x^2 + y^2} \quad (2) z_t = f_x(x, y) g'(t) + f_y(x, y) h'(t)$$

$$2.2 \quad (1) z_u = 10u, z_v = 6v \quad (2) z_u = \frac{u \tan^{-1} \frac{u}{v} + v \log \sqrt{u^2 + v^2}}{u^2 + v^2}, z_v = \frac{v \tan^{-1} \frac{u}{v} - u \log \sqrt{u^2 + v^2}}{u^2 + v^2}$$

$$2.3 \quad \text{略}$$

$$2.4 \quad \text{略}$$

2.5 略

$$2.6 \quad z_u = \frac{5z_x + 4z_y}{22}, \quad z_v = \frac{3z_x - 2z_y}{22}$$

2.7 略

2.8 略

6.3 連続と偏微分

$$3.1 \quad (1) f_x(0,0) = 1 \quad (2) \text{不連続}$$

$$3.2 \quad (1) -b \quad (2) a = b$$

3.3 略

$$3.4 \quad \text{不連続であるが, 偏微分可能で } f_x(0,0) = 0, f_y(0,0) = 0$$

$$3.5 \quad (1) (x^2 + y^2)^2 f_x = y(y^2 - x^2)(e^{x^2} - e^{y^2}) + 2x^2 y(x^2 + y^2)e^{x^2}, \\ (x^2 + y^2)^2 f_y = x(x^2 - y^2)(e^{x^2} - e^{y^2}) - 2xy^2(x^2 + y^2)e^{y^2} \\ (2) f_x(0,0) = 0, f_y(0,0) = 0 \quad (3) f_{xy}(0,0) = -1, f_{yx}(0,0) = 1$$

6.4 偏微分の応用1(最大・最小)

$$4.1 \quad (1) \left(-\frac{3}{4}, -\frac{3}{4}\right) \text{ で, 極小 } -\frac{27}{256}, \left(\frac{9}{4}, -\frac{9}{4}\right) \text{ で, 極小 } \frac{2187}{256} = 3^7 \cdot 2^{-8} \quad (2) \text{ 極値なし} \\ (3) \text{ 極値なし} \quad (4) (\pm\sqrt{2}, \pm\sqrt{2}) \text{ で, 極小 } -8 \quad (5) (0,0) \text{ で, 極大 } 0, (\pm 1, 0) \text{ で極小 } -1 \\ (6) (-1, -1) \text{ で, 極大 } 1 \quad (7) \left(\pm\frac{1}{2}, \pm\frac{1}{2}\right) \text{ で, 極大 } \frac{1}{8}, \left(\pm\frac{1}{2}, \mp\frac{1}{2}\right) \text{ で, 極小 } -\frac{1}{8} \\ (8) a < 0 \text{ のとき, } (-2a, -a) \text{ で, 極大値 } 32a^3, a > 0 \text{ のとき, } (-2a, -a) \text{ で, 極小値 } 32a^3, \\ a = 0 \text{ のとき, 極値なし} \quad (9) (2, 1) \text{ で, 極小 } -4 \quad (10) (0, -4) \text{ で, 極大 } 0, (-1 - \sqrt{3}, 3 + \sqrt{3}) \\ \text{で, 極大 } 4\sqrt{3} + 16 \quad (11) (0, \sqrt{3}) \text{ で, 極大 } 6\sqrt{3} \quad (12) \left(\frac{5}{3}, \frac{1}{3}\right) \text{ で, 極小 } -\frac{13}{3} \\ (13) (\pm\sqrt{3}, \mp\sqrt{3}) \text{ で, 極小 } -18 \quad (14) \left(\frac{1}{\sqrt{3}}, 0\right) \text{ で, 極大 } \frac{2}{3\sqrt{3}}, \left(-\frac{1}{\sqrt{3}}, 0\right) \text{ で, 極小 } -\frac{2}{3\sqrt{3}}$$

$$4.2 \quad (1) (0,0) \text{ で極小 } 0, (0, \pm 1) \text{ で極大 } 2e^{-1} \\ (2) \left(\frac{1}{\sqrt{2}}, 0\right) \text{ で, 極大 } \frac{1}{\sqrt{2}e}, \left(-\frac{1}{\sqrt{2}}, 0\right) \text{ で, 極小 } -\frac{1}{\sqrt{2}e} \quad (3) x = y = \frac{\pi}{3} \text{ で, 極大 } \frac{3\sqrt{3}}{2}$$

$$4.3 \quad (1) F(x, y) = 3(x^2 - y)h + 3(y^2 - x)k + 3(xh^2 - hk + yk^2) \quad (2) x = y = 1 \text{ で, 極小 } -1$$

$$4.4 \quad x = 2n\pi + \frac{\pi}{3}, y = 2m\pi + \frac{\pi}{3} \text{ のとき, 極大 } \frac{3\sqrt{3}}{2}, \\ x = 2n\pi - \frac{\pi}{3}, y = 2m\pi - \frac{\pi}{3} \text{ のとき, 極小 } -\frac{3\sqrt{3}}{2}, \text{ ただし, } m, n \in I$$

$$4.5 \quad (1) y' = -\frac{2x+y}{x+2y}, y'' = -\frac{18}{(x+2y)^3} \quad (2) x = 1 \text{ のとき, 極小 } -2, x = -1 \text{ のとき, 極大 } 2$$

$$4.6 \quad (1) a > 0 \text{ のとき, 極小 } -a^3, a < 0 \text{ のとき, 極大 } -a^3, a = 0 \text{ のとき, 極値なし} \\ (2) x = \sqrt[3]{2} \text{ のとき, 極大 } y = \sqrt[3]{4}$$

$$4.7 \quad (1) \text{ 略} \quad (2) \text{ 略} \quad (3) \text{ 略}$$

4.8 $x = y = -\frac{1}{2}$ のとき , 極小

4.9 $b^2 \leq 3ac$ か $b < 0, c = 0$ か $a \neq 0, b = c = 0$

6.5 偏微分の応用 2(最大・最小)

5.1 (1) $x = y = z = 3$ のとき , 極大 27

(2) $x = y = \pm \frac{1}{\sqrt{2}}$ のとき , 極大 2 , $x = -y = \pm \frac{1}{\sqrt{2}}$ のとき , 極小 0

5.2 最大値 $\frac{5}{3}$, 最小値 -1

5.3 $x = y = z = \frac{1}{3}$ で , 最大値 $\log_2 3$

5.4 $\left(\frac{4}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ で最大値 $\sqrt{6}$, $\left(-\frac{4}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$ で最小値 $-\sqrt{5}$

5.5 $m^m n^n p^p a^{m+n+p} (m+n+p)^{-(m+n+p)}$

5.6 略

6.6 偏微分の応用 (図形)

6.1 (1) $x = y = 1$ のとき , 極小 -1

(2) $A(1, -1)$, $B(1, 1)$, $C(-1, 1)$, $D(-1, -1)$ とすれば , AB 上では $y^3 - 3y + 1$, BC 上では $x^3 - 3x + 1$, CD 上では $y^3 + 3 - 1$, DA 上では $x^3 + 3x - 1$ で表される .

特に $f(A) = 3$, $f(B) = -1$, $f(C) = 3$, $f(D) = -5$

(3) $(-1, -1)$ で最小値 -5 , $(1, -1)$, $(-1, 1)$ で最小値 3

6.2 $x = \frac{\sum_{k=1}^n x_k}{n}$, $y = \frac{\sum_{k=1}^n y_k}{n}$

6.3 底面の半径 : 高さ = $\sqrt[3]{4} : 2\sqrt{\sqrt[3]{5} - \sqrt[3]{4}}$

6.4 $\frac{4S^2}{9abc}$, ただし , S は三角形の面積

6.5 (1) $0 < x \leq 4$ (2) 4

6.6 $6^{-3/2}$

6.7 $\angle B = 150^\circ$, $50 + \frac{125\sqrt{3}}{4}$

6.8 $-\frac{a^2}{2}$

6.9 $\frac{2k^2}{3}$

6.10 $P(x, y)$ は重心

6.11 略

6.12 0

6.7 テイラー展開

7.1 (1) $1 - \frac{x^2 + y^2}{2} - \frac{(x^2 + y^2)^2}{8} - \cdots + (-1)^n \binom{1/2}{n} (x^2 + y^2)^n + \cdots$

(2) $1 + ax + \frac{(a^2 - b^2)x^2}{2!} + \frac{(a^3 - 3ab^2)x^3}{3!} + \cdots + R_n x^n + \cdots$ ただし, $R_n = \frac{1}{n!} \sum_{r=0}^n (-1)^r {}_n C_{2r} a^{n-2r} b^{2r}$

7.2 略

6.8 偏微分の総合問題

8.1 (1) $y' = \frac{x+y}{x-y}$ (2) $y'' = \frac{2(x^2 - 2xy - y^2)}{(x-y)^3}$ (3) 右図

第7章 重積分法

7.1 重積分の計算

1.1 (1) $\pi - 2$ (2) $\frac{1}{2}$ (3) $\frac{4}{3}$

1.2 (1) $\int_0^1 dy \int_0^{2\sqrt{y}} f(x, y) dx + \int_1^3 dy \int_0^{3-y} f(x, y) dx$

1.3 (1) $\frac{\pi}{8}$ (2) $\frac{1}{6}$ (3) $\frac{101}{32} - \log 4$ (4) $\frac{5}{6}$ (5) $\frac{62\sqrt{2}}{35}$ (6) $\frac{5}{3}$ (7) $\frac{4(3-2\sqrt{2})}{5}$ (8) $\frac{1}{15}$ (9) $\frac{11}{3}$
(10) $\frac{1}{6}$

1.4 (1) $\frac{8}{15}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi a^4}{2}$ (4) π (5) $\frac{1}{8}$ (6) $\pi(1-e^{-1})$ (7) 0 (8) 4 (9) $\frac{\pi}{4}$ (10) $\frac{\pi a^6 \left(\frac{3}{p^2} + \frac{1}{pq} + \frac{3}{q^2} \right)}{24}$
(11) 1

1.5 (1) $\frac{(8-3\sqrt{3}-2\pi)a^3}{9}$ (2) $\frac{\pi(2\log 2-1)}{4}$ (3) 2π (4) $\frac{8+\pi}{32}$ (5) $2\pi(\sqrt{2}-1)$ (6) $\frac{32}{9}$ (7) $\frac{16(3\pi-2)}{9}$
(8) $3(3\pi-4)$ (9) 2π (10) $\frac{a^4}{8}$

1.6 (1) $\frac{4a^3}{3}$ (2) $\frac{65}{2}$ (3) $\frac{\pi}{2} - 1$ (4) $1 - \cos 1$

1.7 (1) $\frac{\pi(a^2+b^2)}{4}$

1.8 (1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) 4 (4) $\frac{e-1}{4}$

1.9 (1) $\frac{4\pi c}{3}$

7.2 重積分

2.1 (1) 略 (2) $\frac{R^2}{2} - \pi R \log R$

2.2 $\frac{4a^4}{3}$

2.3 $\frac{2}{3}$

2.4 (1) r (2) $\frac{2\pi a^2}{3}$

2.5 $\frac{2}{3}$

2.6 右図 $\frac{\pi(2\log 2-1)}{4}$

$$2.7 \int_0^t f(x, t) dx + \int_0^t f(t, y) dy$$

$$2.8 (1) \iint_{D'} f(au, bv) ab du dv \quad D': u^2 + v^2 \leq 1 \quad (2) \frac{\pi ab(a^2 + b^2)}{5}$$

$$2.9 (1) \text{右図} \quad (2) e^u \quad (3) \frac{e^3 - 1}{6}$$

$$2.10 (1) \iint_{D'} \frac{(1-r^2)r}{1+r^2} dr d\theta \quad (2) \frac{\pi(2 \log 2 - 1)}{4}$$

$$2.11 (25\sqrt{5} - 31)120$$

7.3 広義積分

$$3.1 (1) \frac{\pi}{2} + \frac{1}{2 \log 2} \quad (2) 8 - 4\sqrt{2} \quad (3) \frac{\pi}{2}$$

$$3.2 (1) \frac{\pi^2}{8 \log 2} \quad (2) \frac{1}{2 \sin 1}$$

$$3.3 (1) \frac{\pi}{4} \quad (2) \pi$$

$$3.4 p < 1 \text{ なら } \frac{\pi a^{2-2p}}{1-p}, p \geq 1 \text{ なら発散}$$

$$3.5 (1) x \sin^{-1} \frac{x}{a} - \sqrt{a^2 - x^2} \quad (2) \pi(f(a) - f(0))$$

$$7.4 \int_0^\infty e^{-x^2} dx$$

$$4.1 (1) \frac{\pi}{4} \quad (2) \frac{\sqrt{\pi}}{2} \quad (3) \frac{\sqrt{\pi}}{2} \quad (4) \pi \quad (5) \frac{\sqrt{pi}}{2}$$

$$4.2 (1) \text{略} \quad (2) \text{略}$$

$$4.3 (1) \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)n!}, \text{収束半径 } \infty \quad (2) 0.8$$

$$4.4 (1) \frac{\iint_{R^2} e^{-(x^2+y^2)/2} dx dy - \iint_{D'} e^{-(x^2+y^2)/2} dx dy}{2\pi} \quad (2) \frac{e^{-a^2/4}}{2\sqrt{\pi}}$$

$$4.5 \frac{\pi}{4}$$

$$4.6 (1) \frac{\pi(1 - e^{-R^2})}{2} \quad (2) I(R) < \iint_D e^{r^2} dS < I(\sqrt{2}R) \quad (3) \frac{\sqrt{\pi}}{2}$$

7.5 重積分の応用 (体積)

$$5.1 (1) \frac{\pi^2}{16} \quad (2) (3 - \sqrt{5})\pi abc \quad (3) \frac{2a^3}{3} \quad (4) \frac{2}{9} \quad (5) \frac{9\pi}{2} \quad (6) \frac{\pi(a^2 + b^2)}{4a^2b^2} \quad (7) \frac{8(3\sqrt{2} - 2)a^3}{3} \\ (8) \frac{\pi(4 - \sqrt{2})a^3}{3}$$

$$5.2 \quad (1) \frac{\pi}{5} \quad (2) 2\pi(2\sqrt{2}-1) \quad (3) 4 \quad (4) \frac{\pi(6\sqrt{3}-8)}{9\sqrt{3}} \quad (5) \frac{2\sqrt{2}a^3}{3} \quad (6) \frac{4\pi abc}{3} \quad (7) \frac{2(3\pi-4)a^3}{3} \\ (8) \frac{2}{3}$$

$$5.3 \quad V = \frac{\pi}{3(a^5 - 3a^4 + 2a^2)} \quad a = \frac{\sqrt{105}-5}{10}$$

$$5.4 \quad (1) \text{右図} \quad (2) \frac{\pi(2\log 2 - 1)}{4}$$

$$5.5 \quad \text{右図} \quad \frac{\pi}{32}$$

$$5.6 \quad 2\pi a^2(1-2a)$$

$$5.7 \quad h = 1.5a$$

$$5.8 \quad \frac{16a^3}{3}$$

$$5.9 \quad \frac{2a^2h(3\pi-2)}{3}$$

$$5.10 \quad (1) \text{右下図} \quad (2) \frac{\pi a^4}{3}$$

$$5.11 \quad (1) \text{回転楕円体: } a^2x^2 + a^2y^2 + (a^2-1)z^2 = a^2(a^2-1), \text{図略} \quad (2) \frac{4\pi}{3a(a^2-1)}$$

7.6 重積分の応用 (面積)

$$6.1 \quad (1) \frac{\pi(\sqrt{2} + \log(1 + \sqrt{2}))}{4} \quad (2) 2\pi a(a-b) \quad (3) 4\pi^2 ab \quad (4) \text{積分を } a, b, c \text{ で直接表すことができない}$$

$$6.2 \quad \alpha: \text{鈍角のとき}, r = \frac{a\left(\tan \frac{\alpha}{2} + \cot \frac{\alpha}{2}\right)}{2}, b = \frac{a\left(\tan \frac{\alpha}{2} - \cot \frac{\alpha}{2}\right)}{2} \text{ とすれば, } S = 4\pi \left(ar - br \sin^{-1} \frac{a}{r}\right) \\ \alpha: \text{鋭角のとき}, r = \frac{a\left(\tan \frac{\alpha}{2} + \cot \frac{\alpha}{2}\right)}{2}, b = \frac{a\left(\cot \frac{\alpha}{2} - \tan \frac{\alpha}{2}\right)}{2} \text{ とすれば, } S = 4\pi \left(r^2 + br \sin^{-1} \frac{a}{r} + r(r-a)\right)$$

$$6.3 \quad (1) abu \quad (2) \pi ab$$

$$6.4 \quad 2\pi a(a-b)$$

$$6.5 \quad 2(\pi + 1 + \sqrt{2})$$

$$6.6 \quad \text{略}$$

7.7 重積分の応用 (物理)

$$7.1 \quad \left(\frac{1}{2}, \frac{2}{5}\right)$$

$$7.2 \quad \left(0, 0, \frac{a}{2}\right)$$

$$7.3 \quad \text{半球体の中心より } \frac{3a}{4} \text{ の所}$$

7.8 重積分の総合問題

8.1 (1) 右上図 (2) a^3b (3) $\frac{\pi a^2 \sqrt{1+b^2}}{2}$ (4) $a^3 = V_0$, $b = 1$

8.2 (1) $0, 2g(x)$ (2) 4 (3) 2π

第8章 関数方程式

8.1 一階微分方程式

$$1.1 \quad (1) 3(x+1)^2 y^2 = 4x(x+2) \quad (2) y = (4x+1)^2 \quad (3) y = C e^{\sin x} \quad (4) 2e^{-x} + e^{-2y} = C$$

$$1.2 \quad (1) x^2 + y^2 = Cx^3 \quad (2) y = x + Cx^2$$

$$(3) |k| < 2 \text{ のとき, } \sqrt{4-k^2} \log(x^2 - kxy + y^2) + 2k \tan^{-1} \frac{2y - kx}{x\sqrt{4-k^2}} = C,$$

$$|k| > 2 \text{ のとき, } \sqrt{k^2-4} \log(x^2 - kxy + y^2) + k \log \frac{2y + \sqrt{k^2-4}x}{2y + \sqrt{k^2-4}} = C,$$

$$|k| = 2 \text{ のとき, } (y \pm kx) \log(x^2 - kxy + y^2) - kx = C(y \pm kx)$$

$$(4) x(3y^2 - x^2) = C$$

$$1.3 \quad (1) y = C e^{2x} - e^x \quad (2) 4y = 2x \log x - x \quad (3) y = \frac{C - \cos \frac{2x}{4}}{\cos x} \quad (4) y = C \sqrt{x^2 + 1} + x$$

$$(5) y = C e^{-x^2/2} + 1 \quad (6) y = \frac{x^3}{2} + x \quad (7) y = C x^{-3x} + \frac{9x^2 - 6x + 11}{27}$$

$$1.4 \quad (2y - (1 - \sqrt{5})x)(y^2 - xy - x^2)^{\sqrt{5}}(2y - (1 + \sqrt{5})x) = C$$

$$1.5 \quad (1) y = \frac{2}{Cx - x^3} \quad (2) y^2 = \frac{1}{x^2 + Cx} \quad (3) y = \frac{2(x-1)}{C + (x-1)^2} \quad (4) y = \frac{e^x}{C + (1-x)e^x}$$

$$1.6 \quad (1) y = \frac{1}{1 - C e^{-x}} \quad (2) y = \frac{1 - e^{y_0}}{1 - e^{y_0} - (e - e^{y_0})e^{-x}}$$

$$1.7 \quad (1) y = C e^{-t} \quad (2) y_0 = \frac{\cos \omega t + \omega \sin \omega t}{1 + \omega^2} \quad (3) y = C e^{-t} + \frac{\cos \omega t + \omega \sin \omega t}{1 + \omega^2}$$

$$(4) y_0 = \frac{7 \sin t + \cos t - 6 \sin^3 t + 2 \cos^3 t}{5}$$

$$1.8 \quad (2x + y + 3) \log(2x + y + 3) = 3x + y + C$$

$$1.9 \quad \text{一般解: } y^2 = C^2 x^2 - (1 + C^2)^2, \quad \text{特殊解: } 4y = \pm(x^2 - 4)$$

$$1.10 \quad (1) \text{ 略} \quad (2) y = 2x - 2 \tan^{-1} x + C$$

$$1.11 \quad k = 1, \text{ 右図}, y = 2e^{-x}$$

$$1.12 \quad (1) u' = u^2 + 5 \quad (2) y = -4x - 2 + \frac{\tan 2x + C}{1 - C \tan 2x} \quad (3) y = -4x - 2 + \frac{\tan 2x + 2}{1 - 2 \tan 2x}$$

$$1.13 \quad y'' + m^2 y = 0$$

$$1.14 \quad (1) x u' - u = -1 \quad (2) \frac{du}{dt} - u = -1 \quad (3) y = x - \frac{x^2}{3}$$

$$1.15 \quad \text{楕円} \quad C^2(x-1)^2 + y^2 = 1$$

$$1.16 \quad y'' = f_x + f_y f, \quad y''' = f_{xx} + 2f_{xy} f + f_{yy} f^2 + (f_x + f_y)^2$$

1.17 $C = -1$ のとき, $y = -1$, $C = 0$ のとき, $y = \frac{1 - e^{2x}}{1 + e^{2x}}$, $C = 1$ のとき, $y = 1$,
 $C = 2$ のとき, $y = \frac{3 + e^{2x}}{3 - e^{2x}}$

1.18 略

1.19 (1) $2xy' + 1 - u = 0$ (2) $x(1 + xy) = C(1 - xy)$

8.2 定係数の線形微分方程式 (1)

2.1 (1) $y = C_1 e^{3x} + C_2 \cos 2x + C_3 \sin 2x$ (2) $a \neq 0$ のとき, $y = C_1 e^{\frac{x}{a}} + C_2 e^{-\frac{x}{a}} + C_3 x + C_4$
 $a = 0$ のとき, $y = C_1 x + C_2$ (3) $y = C_1 e^{-x} + e^{\frac{x}{2}} \left(C_2 \cos \frac{\sqrt{7}x}{2} + C_3 \sin \frac{\sqrt{7}x}{2} \right)$
(4) $y = C_1 e^{2x} + C_2 e^x$ (5) $y = C_1 e^{\alpha x} + C_2 e^{\beta x}$ $\alpha, \beta = \frac{5 \pm \sqrt{5}}{2}$ (6) $y = C_1 e^{-3x} + C_2 e^x$

2.2 (1) $y = e^{-x} (C_1 \cos \sqrt{3}x + C_2 \sin \sqrt{3}x)$ (2) $y = 4e^{-t} - 3e^{-2t}$
(3) $y = \frac{(1 + 2a)e^{2ax}}{4a^2} + \frac{2a^2 x^2 - 2ax - 1 - 2a}{4a^2}$ (4) $y = 3e^{-x} - 2e^{-2x}$

2.3 (1) $y = C_1 \cos kx + C_2 \sin kx$ (2) $k = \frac{\pi}{4} + 2n\pi, \frac{3\pi}{4} + 2n\pi$

2.4 $y = 2e^{-x} \left(\cos \left(2x + \frac{\pi}{6} \right) \right)$ 右図

2.5 0

8.3 定係数の線形微分方程式 (2)

3.1 (1) $y_0 = \frac{2x - 3}{4}$

3.2 (1) $y = C_1 e^{\alpha x} + C_2 e^{\beta x} + \frac{(x-1)e^{2x}}{5}$, α, β は $t^2 + t - 1 = 0$ の解
(2) $y = e^x (C_1 \cos x + C_2 \sin x) + \frac{x^2}{2} + x + 1$ (3) $y = C_1 \sin x + C_2 \cos x + \frac{e^x}{2}$
(4) $y = e^{-x} (C_1 \sin \sqrt{2}x + C_2 \cos \sqrt{2}x) + \frac{3x-2}{9}$
(5) $y = C_1 e^{\alpha x} + C_2 e^{\beta x} + e^{-2x}$ α, β は $t^2 + t - 1 = 0$ の解
(6) $y = e^{-\frac{x}{2}} (C_1 \sin \sqrt{3}x + C_2 \cos \sqrt{3}x) + \frac{e^x}{3}$ (7) $y = e^{-2x} (C_1 \sin 3x + \cos 3x) + \frac{e^{4x}}{5}$
(8) $y = C_1 e^{2x} + C_2 e^{4x} + \frac{e^x}{3}$ (9) $y = C_1 e^{2x} + C_2 e^{-4x} + \frac{x e^{2x}}{6}$ (10) $y = C_1 e^x + C_2 e^{2x} - \frac{x}{2} - \frac{1}{4}$
(11) $y = C_1 e^{2x} + C_2 e^{-x} - \frac{x^2 - 1}{2}$ (12) $y = C_1 \sin x + C_2 \cos x + \frac{e^x}{2} + 1$

3.3 (1) $y = C_1 e^{2x} + C_2 e^{-3x} - \frac{7 \sin x + \cos x}{8}$
(2) $y = e^{-\frac{x}{2}} \left(C_1 \cos \frac{\sqrt{7}x}{2} + C_2 \sin \frac{\sqrt{7}x}{2} \right) + \sin x - \cos x$
(3) $y = e^x (C_1 \sin 2x + C_2 \cos 2x) + (2 \sin x + \cos x)$ (4) $y = (C_1 x + C_2) e^x + 2 \cos x$
(5) $y = C_1 e^x + C_2 e^{4x} + \frac{(4 - n^2) \cos nx - 5n \sin nx}{n^2(n^2 + 1)(n^2 + 16)}$
(6) $y = e^{-x} (C_1 \sin 2x + C_2 \cos 2x) + \frac{2 \sin x - \cos x}{4}$ (7) $y = C_1 e^{-x} + C_2 e^{-2x} + \frac{\cos x + 3 \sin x}{10}$

$$3.4 \quad (1)y = e^x(C_1 \cos x + C_2 \sin x) + \frac{e^{-x}(\cos x - \sin x)}{8}$$

$$3.5 \quad (1)y = C_1 e^x + C_2 e^{2x} - x e^x \quad (2)y = e^x(C_1 \cos 2x + C_2 \sin 2x) + e^x \sin \frac{2x}{4}$$

$$(3)y = e^{2x}(C_1 \cos 2x + C_2 \sin 2x) + \frac{e^{2x}}{4} \quad (4)y = C_1 e^{2x} + C_2 e^{3x} + \frac{e^x}{2}$$

$$(5)y = C_1 e^x + C_2 e^{-2x} + \frac{x e^x}{4} \quad (6)y = C_1 e^x + C_2 e^{-2x} + \frac{x e^x}{3} \quad (7)y = C_1 e^{4x} + C_2 e^{-2x} - \frac{x e^{-2x}}{6}$$

$$3.6 \quad (1)y = C_1 e^x + C_2 e^{2x} + \frac{2x+3}{4} + \frac{e^{2x}(\sin x - \cos x)}{2}$$

$$(2)y = C_1 e^x + C_2 e^{2x} + \frac{4x^2+6x+5}{8} - \frac{e^x(\sin x - \cos x)}{2}$$

$$(3)y = e^x(C_1 \cos x + C_2 \sin x) + \sin x + \cos x \quad (4)x = C_1 e^{2t} + C_2 e^{-t} + \frac{x e^{2t}}{3} + \frac{\cos x - 3 \sin x}{10}$$

$$3.7 \quad (1)y = C_1 e^x + C_2 e^{-x} + C_3 e^{2x} + x e^{2x} \quad (2)y = C_1 e^x + C_2 e^{-3x} + C_3 + \frac{2x^2+4x+5}{8} - e^x \sin \frac{2x}{20}$$

$$(3)y = (C_1 x + C_2) e^{2x} + C_3 + \frac{x^2 e^{2x}}{4} \quad (4)y = C_1 e^x + C_2 e^{-x} + C_3 e^{2x} x e^{2x}$$

$$3.8 \quad (1)y = e^{-\frac{x}{2}} \left(C_1 \sin \frac{\sqrt{3}x}{2} + C_2 \cos \frac{\sqrt{3}x}{2} \right) \quad (2)y = 3e^{-\frac{x}{2}} + e^{-2x} + \frac{1}{2} \quad (3)y = 2e^{-2x} - 3e^{-x} + 1$$

$$(4)u = (a+b-1)e^x + \left(\frac{a-b}{2} + \frac{1}{6} \right) e^{-x} + \frac{e^{2x}}{3}$$

$$3.9 \quad \alpha \neq \beta \text{ のとき } , y = C_1 e^{\alpha x} + e^{-\xi x} (C_2 \cos \sqrt{1-\xi^2} x + C_3 \sin \sqrt{1-\xi^2} x) + \frac{e^{\beta x}}{(\beta-\alpha)(\beta^2+2\xi\beta+1)},$$

$$\alpha = \beta \text{ のとき } , y = C_1 e^{\alpha x} + e^{-\xi x} (C_2 \cos \sqrt{1-\xi^2} x + C_3 \sin \sqrt{1-\xi^2} x) + \frac{x e^{\beta x}}{\beta^2+2\xi\beta+1}$$

$$3.10 \quad (1)\alpha < 2 \quad (2)y = \sqrt{2} \sin \left(\sqrt{2}x + \frac{\pi}{4} \right), \text{ 図略}$$

$$3.11 \quad (1)a \neq \pm 1 \text{ のとき } , y = C_1 \sin x + C_2 \cos x + \sin \frac{ax}{1-a^2},$$

$$a = \pm 1 \text{ のとき } , y = C_1 \sin x + C_2 \cos x - ax \cos \frac{x}{2} \quad (2)y_0 = -x \cos \frac{x}{2}$$

$$3.12 \quad u'' + 2u' = 0 \quad y = \frac{e^x - e^{-x}}{2} + \sin x$$

$$3.13 \quad (1)z'' - 6z' + 6z = \log x$$

$$(2)z = C_1 e^{(3+\sqrt{3})x} + C_2 e^{(3-\sqrt{3})x} + e^{(3+\sqrt{3})x} \int e^{(-3-\sqrt{3})x} \log x \frac{dx}{2\sqrt{3}} - e^{(3-\sqrt{3})x} \int e^{(-3+\sqrt{3})x} \log x \frac{dx}{2\sqrt{3}}$$

$$(3)y = C_1 e^{(2+\sqrt{3})x} + C_2 e^{(2-\sqrt{3})x} + e^{(2+\sqrt{3})x} \int e^{(-3-\sqrt{3})x} \log x \frac{dx}{2\sqrt{3}} - e^{(2-\sqrt{3})x} \int e^{(-3+\sqrt{3})x} \log x \frac{dx}{2\sqrt{3}}$$

8.4 二階微分方程式

$$4.1 \quad (1)x^2 > 1 \text{ のとき } y = ax + C_1 \left(x\sqrt{x^2-1} - \log |x + \sqrt{x^2-1}| \right) + C_2$$

$$x^2 < 1 \text{ のとき } y = ax + C_1 \left(x\sqrt{1-x^2} + \sin^{-1} x \right) + C_2$$

$$4.2 \quad (1)y = C_1 x + \frac{C_2}{x^3} - \log x \quad (2)y = C_1 \cos(\log x) + C_2 \sin(\log x)$$

$$(3)y = \log x + C_1 \cos(\log x) + C_2 \sin(\log x) \quad (4)y = \frac{C_1 + C_2 \log x}{x} + \frac{x}{4}$$

$$(5)y = C_1 \cos(\log x) + C_2 \sin(\log x) + \frac{x}{2} \quad (6)y = C_1 \log x + C_2$$

$$4.3 \quad (1)y^2 = a^2x^2 + y_0^2 \quad (2)y^3 = C_1e^{2x} + C_2 \quad (3)y^2 = C_1x + C_2$$

$$4.4 \quad (1) \text{略} \quad (2) \text{略} \quad (3) \text{略}$$

$$4.5 \quad (1)2 \quad (2)x(x+1)u'' + (2x+4)u' = 0 \quad (3)y = C_1 \left(x + 1 + \frac{x}{3} \right) + C_2x^2$$

$$4.6 \quad y = C_1 \cos(\sin x) + C_2 \sin(\sin x)$$

8.5 微分方程式の応用 (図形)

$$5.1 \quad (1)xy' = 2y \quad (2)x^2 = 4y$$

$$5.2 \quad ky' = \sqrt{y^2 - k^2}, \quad y = \frac{\sqrt{x^2 + k^4}}{k}$$

$$5.3 \quad y = x \log x + cx$$

$$5.4 \quad (1)b = \pm\sqrt{a^2 \pm k^2}(a^2 + 1) \quad (2)x^2 + 2y^2 = 2$$

$$5.5 \quad (x-a)^2 + (y-b)^2 = r^2 \quad r : \text{任意の定数}$$

$$5.6 \quad (1)xy' + y = 0 \quad (2)xy = \sqrt{21} \quad (3)\sqrt{21}\pi \left(\frac{2}{5} + \log 3 \right)$$

$$5.7 \quad (1) \text{右図} \quad (2)y = 2xy' \quad (3)2x^2 + y^2 = C^2$$

$$5.8 \quad 2x^2 + y^2 = 2r^2$$

$$5.9 \quad a \log \frac{a - \sqrt{a^2 - y^2}}{y} - \sin^{-1} \frac{y}{a} = x + C$$

$$5.10 \quad (1)(\sqrt{3}x - y)y' = \sqrt{3}y + x \quad (2)\frac{dr}{d\theta} = \sqrt{3}r \quad (3)x^2 + y^2 = \exp \left(2\sqrt{3} \left(\tan^{-1} \frac{y}{x} - \frac{\pi}{2} \right) \right)$$

$$5.11 \quad (1)y^2 = 2ax + C \quad (2)y^2 = 2a(x-1)$$

$$5.12 \quad (1)2xyy' = y^2 - x^2 \quad (2)y^2 = Cx^2(x^2 + y^2)$$

$$5.13 \quad (1)x - x_0 + a(y - y_0) = 0 \quad (2)(x-1)^2 + (y-2)^2 = r^2 \quad (3)(x-1)^2 + (y-2)^2 = \frac{1}{10}$$

8.6 微分方程式の応用 (現象)

$$6.1 \quad R = R_0 \sqrt{1 - \frac{t}{T}}$$

$$6.2 \quad y(t) = \frac{aNe^{akt}}{a - N + Ne^{akt}}$$

$$6.3 \quad k^2 > 4mR \text{ のとき, } x = Ae^{-\alpha t} + Be^{-\beta t}, \text{ ただし, } \alpha, \beta \text{ は } mt^2 + kt + R = 0 \text{ の解}$$

$$k^2 = 4mR \text{ のとき, } x = (Ax + B)e^{-\alpha t}, \text{ ただし, } \alpha = \frac{k}{2m}$$

$$k^2 < 4mR \text{ のとき, } x = e^{-\frac{kt}{2m}} \left(A \sin \sqrt{4mR - k^2}t + B \cos \sqrt{4mR - k^2}t \right)$$

8.7 級数による解法

$$7.1 \quad (1)(n+1)(n+2)a_{n+2} = a_n \quad (2)a_{2n} = \frac{2}{(2n)!}, \quad a_{2n+1} = 0$$

$$7.2 \quad (1)c_n = \prod_{k=1}^n \frac{(\alpha+k-1)(\beta+k-1)}{k(\gamma+k-1)} \quad (2)1$$

$$7.3 \quad (1)(n+1)a_{n+1} = a_n \quad (2) \text{略}$$

8.8 連立微分方程式

$$8.1 \quad (1)x = C_1 e^t + 5C_2 e^{4t}, \quad y = C_1 e^t + 2C_2 e^{4t} \\ (2)x = 2e^{3t}(A \sin t + B \cos t), \quad y = e^{3t}((A-B) \sin t + (A+B) \cos t) \\ (3)y = 2e^{5t}(A \sin t + B \cos t), \quad z = e^{5t}((A-B) \cos t + (A+B) \sin t)$$

$$8.2 \quad (1)x = \frac{e^{3t} + e^{-t}}{2}, \quad y = \frac{e^{3t} - e^{-t}}{2}$$

$$8.3 \quad (1) \text{略} \quad (2)2x = 2 \cos 4t - \sin 4t, \quad 2y = 2 \cos 4t + \sin 4t$$

$$8.4 \quad (1)y = C_1 e^{-2t} + C_2 e^{3t} \quad (2)y = y_1, \quad y' = y_2 \text{ とおくと, } y_1' = y_2, \quad y_2' = 6y_1 + y_2 \\ (3)y = -\frac{e^{-2t}}{5} + \frac{e^{3t}}{5}$$

$$8.5 \quad (1)f(x) = \frac{e^x + e^{-x}}{2}, \quad g(x) = \frac{e^x - e^{-x}}{2} \\ (2)n: \text{奇数のとき } f^{(n)}(x) = \frac{e^x - e^{-x}}{2}, \quad g^{(n)}(x) = \frac{e^x + e^{-x}}{2} \\ n: \text{偶数のとき } f^{(n)}(x) = \frac{e^x + e^{-x}}{2}, \quad g^{(n)}(x) = \frac{e^x - e^{-x}}{2} \\ (3) \text{略}$$

8.9 行列微分方程式

$$9.1 \quad (1)\frac{2}{3} \quad (2)1 \quad (3)\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad (4)\begin{pmatrix} -3e^t + 4e^{2t} \\ -2e^t + 4e^{2t} \end{pmatrix}$$

$$9.2 \quad (1)\begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} \quad (2) \text{固有値 } 2, 3, \text{ 固有ベクトル } t \begin{pmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \\ (3) \text{略} \quad (4)u' = 2u, \quad v' = 3v \quad u = C_1 e^{2t}, \quad v = C_2 e^{3t} \\ (5)x = 2C_1 e^{2t} + C_2 e^{3t}, \quad y = -C_1 e^{2t} - C_2 e^{3t}$$

8.10 積分方程式

$$10.1 \quad a = \frac{1}{2}, \quad b = 0$$

$$10.2 \quad (1)y'' + 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = -1 \quad (2)y = e^{-t}$$

$$10.3 \quad y_n(x) = \sum_{k=0}^n \frac{x^{2k}}{k!}, \quad y(x) = e^{x^2}$$

8.11 差分方程式

11.1 $f(t) = 2^{n+2} + 3^n$

8.12 偏微分方程式

12.1 $f(t) = -\log t$

8.13 関数方程式の総合問題

13.1 (1) $a = -1$, $b = 2$ (2) 略 (3) $C_1 = \frac{1}{2}$, $C_2 = 1$ (4) $y\left(\frac{n\pi}{2}\right) = (-1)^n e^{-\frac{n\pi}{2}}$, 下図

13.2 $x = e^{at}$, $y = \frac{a(e^{bt} - e^{-at})}{a+b}$, $\frac{a}{a+b}$