5) テイラー展開

関数 $f(x)$ の $x = a$ 付近でのテイラー展開は、

$$
\begin{aligned}
f(x) &= f(a) + f'(a)(x-a)\frac{1}{2!}f''(a)(x-a)^2 + \cdots \\
&= \sum_{n=0}^{\infty}\frac{1}{n!}f^n(a)(x-a)^n
\end{aligned}
$$

$$(1+x)^k = 1 + kx + \frac{1}{2}k(k-1)x^2 + \cdots (|x| < 1)$$

$$(1-x)^{-p-1} = 1 + (p+1)x + \frac{1}{2}(p+1)(p+2)x^2 + \cdots (|x| < 1)$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \cdots = \sum_{n=0}^{\infty}\frac{1}{n!}x^n$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)^n}$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots$$

$$\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots = \sum_{n=0}^{\infty}(-1)^{n-1}\frac{1}{n}x^n(-1 < x \le 1)$$

$$\tan^{-1}x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots (-1 \le x \le 1)$$

$$\sinh x = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$$

$$\cosh x = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots$$