International Economics B
3. Oligopoly models

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Oligopoly and noncooperative game theory

- **Oligopoly: A market form in which a market or industry is dominated by a small number (two or more) of sellers (oligopolists).**
- **Because there are few sellers, each oligopolist is likely to be aware of the actions of the others.**
  - **Decisions of one firm influence, and are influenced by, the decisions of other firms.**

  $\Rightarrow$ Strategic interaction among firms.
- **Analyzing oligopolistic markets $\leftarrow$ Applying noncooperative game theory**

Types of oligopolistic competition

- **Cournot competition**
  - **Augustin Cournot,** *Researches Into the Mathematical Principles of the Theory of Wealth*, **1838.**
  - **Firms choose outputs.**
- **Bertrand competition**
  - **Joseph Bertrand (1883), "Review of 'Theorie Mathematique de la Richesse Sociale' by Leon Walras and 'Recherches sur les Principes Mathematiques de la Theorie des Richesses' by Augustin Cournot,"** *Journal des Savants* **67, pp. 499-508.**
  - **Firms choose prices.**
- **Other types of competition (investment in R&D, advertisement, etc.)**

Product differentiation

- Most industries produce a large number of similar but not identical products.
- Product differentiation: the process by which a product is distinguished from others, so that it appeals more to the target consumers.
  - Instruments: brand, quality, function, design, advertising, customer service, etc.
- In the presence of product differentiation, if a firm raises the price of its product, the demand for (= sales of) other firms' products can increase.

A model with two products (and a numeraire)

- Quasi-linear utility: $U(q_1, q_2, m) = u(q_1, q_2) + m$
  - q_i: consumption of good i ($i = 1, 2$)
  - m: consumption of numeraire
  - u: twice differentiable, increasing, and strictly concave
- Utility maximization $\max_{q_1,q_2} u(q_1, q_2) - \sum_{i=1,2} p_i q_i$

  $\Rightarrow$ FOCs:

$$u_1(q_1, q_2) = p_1, \quad u_2(q_1, q_2) = p_2$$

  $\Rightarrow$ Demand functions: $q_i = d^i(p_1, p_2)$
- Properties of $d^i(\cdot, \cdot)$
  - Demand curve is downward sloping: $d^i_i < 0$
  - Cross effect d^i_j, $j \neq i$, is positive (negative) for substitutes (complements)
  - Assume that own effect is larger than the cross effect: $|d^i_i| > |d^i_j|$ (or equivalently, $|u_{ii}| > |u_{ij}|$), $j \neq i$

- **Quadratic subutility function:**

$$u(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2}{2}$$

  - **Parameter restrictions:** $\alpha_i, \beta_i > 0$, $\beta_1\beta_2 - \gamma^2 > 0$, $\alpha_i\beta_j - \alpha_j\gamma > 0$, $i, j = 1, 2, j \neq i$
- **FOCs:**

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2, \quad p_2 = \alpha_2 - \gamma q_1 - \beta_2 q_2$$

  - **Inverse demand functions**

$\Rightarrow$ **Direct demands:**

$$q_1 = a_1 - b_1 p_1 + c p_2, \quad q_2 = a_2 + c p_1 - b_2 p_2,$$

  - **where** $a_i = (\alpha_i\beta_j - \alpha_j\gamma)/(\beta_1\beta_2 - \gamma^2)$ **and** $b_i = \beta_j/(\beta_1\beta_2 - \gamma^2)$, $i, j = 1, 2, j \neq i$

- **Properties of demand functions:**
  - **The products are substitutes/independent/complements according to whether $\gamma > 0 / = 0 / < 0$.**
  - **Demand curve is always downward sloping.**
  - **When $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2 = \gamma > 0$, the products are perfect substitutes (i.e., homogeneous).**
  - **When $\alpha_1 = \alpha_2$, $\gamma^2/(\beta_1\beta_2)$ expresses the degree of product differentiation;**
    - **$\gamma^2/(\beta_1\beta_2) \to 0$ when the products are independent.**
    - **$\gamma^2/(\beta_1\beta_2) \to 1$ when the products are homogeneous (provided $\gamma > 0$).**

Static Models of Oligopoly

- **Cournot competition in homogeneous products**
  - **Two firms (duopoly) or n firms**
  - **If $n \to \infty$, a competitive outcome arises.**
- **Bertrand competition in differentiated products**
  - **If products were homogeneous, Bertrand competition leads to a competitive outcome.**

Cournot competition in homogeneous products

- Firms determine their respective outputs.
  - n firms
  - Homogeneous products (i.e., no product differentiation)
- Market clearing condition: $D(p) = \sum_{i=1}^{n} x_i \equiv X$
  - $D(p)$: market demand function, assumed to be twice differentiable and $D' < 0$
  - x_i: output of firm i's product $(i = 1, \cdots, n)$

  $\Rightarrow$ Market price is a function of total outputs X: $p = P(X)$
  - "Inverse demand function", i.e., $P(\cdot) = D^{-1}(\cdot)$
- Firm i's cost function: $C_i(x_i)$, assumed to be twice differentiable, $C_i' > 0$, and $C_i'' \geq 0$

Case of $n = 2$ firms (duopoly)

- **Taking the rival firm's output x_j as given ($j \neq i$), firm i determines x_i so as to maximize its profit:**

$$\pi^i(x_1, x_2) = P(x_1 + x_2)x_i - C_i(x_i), \quad i = 1, 2.$$

- **FOC:**

$$\frac{\partial \pi^i}{\partial x_i} = P(x_1 + x_2) + P'(x_1 + x_2)x_i - C_i'(x_i) = 0$$

$\Rightarrow$ **Firm i's best-response function: $x_i = R_i(x_j)$, $i, j = 1, 2,\ j \neq i$**

Definition

The triplet $\{p^c, x_1^c, x_2^c\}$ is a Cournot-Nash equilibrium if:

1. x_1^c **maximizes** $\pi^1(x_1, x_2^c)$ **and** x_2^c **maximizes** $\pi^2(x_1^c, x_2)$;
2. $p^c = P(x_1^c + x_2^c)$.

- **Using the best-response functions, (x_1^c, x_2^c) must satisfy**

$$x_1^c = R_1(x_2^c) \quad \text{and} \quad x_2^c = R_2(x_1^c).$$

- **Slope of the best-response function:**

$$R_i'(x_j) = -\frac{\pi^i_{ij}(x_1, x_2)}{\pi^i_{ii}(x_1, x_2)} = -\frac{P' + P''x_i}{2P' + P''x_i - C_i''}$$

  - **SOC:** $\pi^i_{ii} < 0$
  - **If** $\pi^i_{ij} < 0$**, the reaction curve is downward sloping ("strategic substitutes").**
- **The Cournot-Nash equilibrium is, if it exists, unique and stable provided that** $\pi^1_{11}\pi^2_{22} > \pi^1_{12}\pi^2_{21}$**.**
  - **Comparison between the slopes of the two firms' reaction curves**

- **Example with linear demand and constant marginal cost**
  - **Market demand function:** $D(p) = a - p$, $a > 0$
    $\Rightarrow$ **Inverse demand function:** $P(X) = a - X$
  - **Cost function:** $C(x_i) = c_i x_i$, $c_i \in [0, a)$
- **Firm** i**'s profit:** $\pi^i(x_i, x_j) = (a - x_1 - x_2)x_i - c_i x_i$
  $\Rightarrow$ **Best-response function:**

$$x_i = R_i(x_j) = \frac{a - c_i - x_j}{2}$$

- **Cournot–Nash equilibrium**
  - **Outputs:**

$$x_1^c = \frac{a - 2c_1 + c_2}{3}, \quad x_2^c = \frac{a + c_1 - 2c_2}{3}$$

  - **Price:**

$$p^c = \frac{a + c_1 + c_2}{3}$$

- **Example with linear demand and constant marginal cost (cont'd)**
  - **Equilibrium profits:**

$$\pi_i^c = \frac{(a - 2c_i + c_j)^2}{9} = (x_i^c)^2, \quad i = 1, 2.$$

  - **Comparative statics:**

$$\frac{\partial x_i^c}{\partial c_i} < 0, \quad \frac{\partial x_j^c}{\partial c_i} > 0, \quad \frac{\partial p^c}{\partial c_i} > 0$$

- **Comparison with monopoly (or collusive outcome)**
  - **Assume $C_i(x_i) = cx_i$, $c \in [0, a)$, $i = 1, 2$, and the monopolist has the same cost function.**
  - **Monopolist's profit: $\pi(X) = P(X)X - cX$ $\Rightarrow$ FOC:**

$$P(X) + P'(X)X = c$$

  - **Monopoly equilibrium: (X^m, p^m)**
  - **Comparison b/w monopoly and duopoly outcomes:**

$$x_1^c + x_2^c > X^m, \quad p^c < p^m$$

  - **Because duopoly is more competitive than monopoly, the dead-weight loss is smaller.**

Case of many firms ($n > 2$)

- **Each oligopolist's FOC:**

$$P(X) + P'(X)x_i = C_i'(x_i), \quad X \equiv \sum_{j=1}^{n} x_j$$

- **LHS can be rewritten as**

$$P(X)\left[1 + \frac{dP}{dX}\frac{X}{P}\frac{x_i}{X}\right] = P(X)\left[1 - \frac{s_i}{\epsilon}\right],$$

  - $\epsilon \equiv -\frac{dD(p)}{dp} \cdot \frac{p}{D(p)} > 0$: **price elasticity of demand**
  - $s_i \equiv x_i/X$: **firm i's share in the market**

- **Assume $C_i(x_i) = C(x_i)$ for all $i = 1, \cdots, n$**
  $\Rightarrow$ Symmetric Cournot–Nash equilibrium with each firm producing the same amount x^c
  - $s_i = 1/n$ **for all** $i = 1, \cdots, n$
- **In the symmetric equilibrium,**

$$p^c \left[1 - \frac{1}{n\epsilon}\right] = C'(x^c)$$

- **If $n \to \infty$, p^c approaches the competitive equilibrium price, which is equal to the marginal cost.**

Bertrand competition in differentiated goods

- Firms determine their respective products' prices
  - 2 firms (duopoly)
  - Products are differentiated.
- Demand function for firm i's product: $d^i(p_1, p_2)$
  - $\partial d^i / \partial p_i < 0$, $\partial d^i / \partial p_j > 0$, $i, j = 1, 2$, $j \neq i$
- Firm i's cost function: $C_i(x_i)$
  - $C_i' > 0$, $C_i'' \geq 0$
- Firm i chooses p_i, taking p_j as given, so as to maximize

$$\pi^i(p_1, p_2) = p_i d^i(p_1, p_2) - C_i(d^i(p_1, p_2)).$$

- **FOC:**

$$\frac{\partial \pi^i}{\partial p_i} = d^i(p_1, p_2) + \left[p_i - C_i'(d^i(p_1, p_2))\right] d_i^i(p_1, p_2) = 0$$

$\Rightarrow$ **Firm i's best-response function: $p_i = R_i(p_j)$, $i, j = 1, 2,\ j \neq i$**

Definition

The pair $\{p_1^b, p_2^b\}$ is a Bertrand-Nash equilibrium if p_1^b maximizes $\pi^1(p_1, p_2^b)$ and p_2^b maximizes $\pi^2(p_1^b, p_2)$.

- **Using the best-response functions, (p_1^b, p_2^b) must satisfy**

$$p_1^b = R_1(p_2^b) \text{ and } p_2^b = R_2(p_1^b).$$

- **Consider linear demand and constant marginal cost:**

$$d^i(p_i, p_j) = A - p_i + bp_j, \quad A > 0,\ 0 < b < 1,$$
$$C_i(x_i) = c_i x_i, \quad c_i \geq 0.$$

  - **A lower b implies a higher degree of product differentiation.**

- **Firm i's best-response function:**

$$p_i = R_i(p_j) = \frac{A + c_i + bp_j}{2}.$$

  - **Reaction curve is upward sloping ("strategic complements").**

- **Bertrand–Nash equilibrium:**

$$p_1^b = \frac{(2+b)A + 2c_1 + bc_2}{4-b^2}, \quad p_2^b = \frac{(2+b)A + bc_1 + 2c_2}{4-b^2}.$$

- **Comparative statics:**

$$\frac{\partial p_i^b}{\partial c_i} > 0, \quad \frac{\partial p_j^b}{\partial c_i} > 0$$

Sequential-move oligopoly

- **Firms may not act simultaneously but sequentially.**
  - **A firm unveils its new product → Rival firms follow up by launching the competing products.**
  - **A firm revises the price of its product → Rival firms also revise the price of the competing product.**
- **Heinrich von Stackelberg,** *Market Structure and Equilibrium*, **1934.**
- **Analysis of the firms' behavior and the outcome of the game:**
  - **Formulate as an extensive-form game: e.g., Firm 1 moves first and then firm 2 makes a decision.**
  - **Derive the subgame perfect equilibrium.**

Preliminary: Isoprofit curves

- **Cournot competition: firm 1's isoprofit curve is the pairs of outputs (x_1, x_2) that yield firm 1 the same profit**
  - $\pi^1(x_1, x_2) = \bar{\pi}_1$, **where** $\bar{\pi}_1$ **is a given constant.**
  - $\Rightarrow \pi^1_1 dx_1 + \pi^1_2 dx_2 = d\bar{\pi}_1$
    - $\pi^1_1 > (=)(<)\ 0$ **if** $x_1 < (=)(>)\ R_1(x_2)$
    - $\pi^1_2 = P'(x_1 + x_2)x_1 < 0$

    $\Rightarrow$ **For a given** x_1**, a higher profit of firm 1 corresponds to a smaller** x_2**.**
  - **Slope of the isoprofit curve:**

$$\left.\frac{dx_2}{dx_1}\right|_{\pi^1(\cdot,\cdot)=\bar{\pi}_1} = -\frac{\pi^1_1}{\pi^1_2} > (=)(<)0 \quad \text{if} \quad x_1 < (=)(>)R_1(x_2)$$

$\Rightarrow$ **The reaction curve intercepts the isoprofit curves where the slope becomes zero (i.e. horizontal).**

- **Bertrand competition: firm 1's isoprofit curve is the pairs of prices (p_1, p_2) that yield firm 1 the same profit**
  - $\pi^1(p_1, p_2) = \bar{\pi}_1 \Rightarrow \pi_1^1 dp_1 + \pi_2^1 dp_2 = d\bar{\pi}_1$
    - $\pi_1^1 > (=)(<)\ 0$ **if** $p_1 < (=)(>)\ R_1(p_2)$
    - $\pi_2^1 = \left[p_1 - C_1'(d^1(p_1, p_2))\right] d_2^1(p_1, p_2) > 0$

    $\Rightarrow$ **For a given p_1, a higher profit of firm 1 corresponds to a higher p_2.**
  - **Slope of the isoprofit curve:**

$$\left.\frac{dp_2}{dp_1}\right|_{\pi^1(\cdot,\cdot)=\bar{\pi}_1} = -\frac{\pi_1^1}{\pi_2^1} < (=)(>) 0 \quad \text{if} \quad p_1 < (=)(>) R_1(p_2)$$

    $\Rightarrow$ **The reaction curve intercepts the isoprofit curves where the slope becomes zero (i.e. horizontal).**

Sequential-move game: Cournot competition

- **Consider a duopoly market and suppose that firm 1 moves first ("leader") and then firm 2 sets its output ("follower").**
- **Deriving the SPE (or Stackelberg equilibrium): Backward induction:**
  1. **(2nd stage) Find the follower's optimal choice for x_2 for each given x_1.**
  2. **(1st stage) Taking the follower's choice into account, find the leader's optimal choice for x_1.**

- **Follower's optimization problem (in the 2nd stage):** **Given x_1, maximize $\pi^2(x_2, x_1)$.** $\Rightarrow$ **Solution: $x_2 = R_2(x_1)$**
- **Leader's optimization problem (in the 1st stage):** **Maximize $\pi^1(x_1, x_2)$ subject to $x_2 = R_2(x_1)$.**
- **FOC:**

$$\frac{d\pi^1(x_1, R_2(x_1))}{dx_1} = \pi^1_1(x_1, R_2(x_1)) + \pi^1_2(x_1, R_2(x_1)) R_2'(x_1) = 0$$

$\Rightarrow$ **Firm 1 chooses the optimal output x_1^s such that its isoprofit curve is tangent to firm 2's reaction curve.**

- **Subgame Perfect Equilibrium:** $(x_1^s, R_2(x_1))$
  - **Pair of the firms' strategy.**
- **Stackelberg equilibrium outcome:** $\{p^s, x_1^s, x_2^s\}$, **where** $x_2 = R_2(x_1^s)$ **and** $p^s = P(x_1^s + x_2^s)$
  - **Outputs that the firms actually set in equilibrium.**
- **Case of linear demand and common constant marginal cost**
  - $P(X) = a - X$, $a > 0$
  - $C_i(x_i) = cx_i$, $c \in [0, a)$, $i = 1, 2$

$\Rightarrow$ **Equilibrium outcome:**

$$x_1^s = \frac{a-c}{2}, \quad x_2^s = \frac{a-c}{4}, \quad p^s = \frac{a+3c}{4}$$

- **Cournot–Nash equilibrium output:** $x_1^c = x_2^c = (a - c)/3$
- **Leader chooses larger output than under the Cournot–Nash equilibrium, while follower chooses smaller output:** $x_1^s > x_1^c = x_2^c > x_2^s$.
  - **Firm 1 knows that firm 2 will reduce its output in response to an increase in firm 1's output.**
  - $\Rightarrow$ **When firm 1 expands output, it expects the price to fall faster under Cournot than under sequential-moves market structure.**
  - $\Rightarrow$ **In order to maintain a high price, firm 1 produces more under the sequential game than under Cournot.**

- **Equilibrium profits:**

$$\pi_1^c = \pi_2^c = \frac{(a-c)^2}{9},$$
$$\pi_1^s = \frac{(a-c)^2}{8}, \quad \pi_2^s = \frac{(a-c)^2}{16}.$$

- **Leader earns higher profits than under the Cournot–Nash equilibrium, while the follower earns lower profits:** $\pi_1^s > \pi_1^c = \pi_2^c > \pi_2^s$
- **Being a leader is better.**

Sequential-move game: Bertrand competition

Assumptions:

- **Bertrand competition: Firms set prices.**
- **Firm 1 moves first ("leader") and then firm 2 sets its price ("follower").**

Deriving the SPE: Backward induction

1. **(2nd stage) Find the follower's optimal choice for p_2 for each given p_1.**
2. **(1st stage) Taking the follower's choice into account, find the leader's optimal choice for p_1.**

- **Follower's optimization problem (in the 2nd stage):**
  Given p_1, maximize $\pi^2(p_2, p_1)$
  $\Rightarrow$ Solution: $p_2 = R_2(p_1)$
- **Leader's optimization problem (in the 1st stage):**
  Maximize $\pi^1(p_1, p_2)$ subject to $p_2 = R_2(p_1)$
- **FOC:**

$$\frac{d\pi^1(p_1, R_2(p_1))}{dp_1} = \pi_1^1(p_1, R_2(p_1)) + \pi_2^1(p_1, R_2(p_1))R_2'(p_1) = 0$$

$\Rightarrow$ Firm 1 chooses the optimal price p_1^s such that its isoprofit curve is tangent to firm 2's reaction curve.

- **Subgame Perfect Equilibrium: $(p_1^S, R_2(p_1))$**

- **Consider a case with linear demand $d^i(p_i, p_j) = A - p_i + bp_j$ and constant marginal cost $C_i(x_i) = c_i x_i$**
  - **Moreover, assume $c_1 = c_2 = 0$.**
- **Equilibrium outcome:**

$$p_1^s = \frac{A(2+b)}{2(2-b^2)}, \quad p_2^s = R_2(p_1^s) = \frac{A(4+2b-b^2)}{4(2-b^2)}$$

- **Follower sets lower price than the leader: $p_1^s > p_2^s$.**

- **In the Bertrand–Nash equilibrium, prices are determined as $p_1^b = p_2^b = A/(2-b)$.**
- **Both leader and follower set higher prices than under the Bertrand–Nash equilibrium: $p_1^s > p_1^b$ and $p_2^s > p_2^b$.**
    - **In the 2nd stage, firm 2 can obtain a larger market share by slightly undercutting p_1.**
    - **$\Rightarrow$ Firm 1 has an incentive to maintain a high price to avoid having firm 2 set a very low price.**

- **Equilibrium profits:**

$$\pi_1^b = \pi_2^b = \frac{A^2}{(2-b)^2},$$
$$\pi_1^s = \frac{A^2(2+b)^2}{8(2-b^2)}, \quad \pi_2^s = \frac{A^2(4+2b-b^2)^2}{16(2-b^2)^2}.$$

- **Both leader and follower earn higher profits than under the Bertrand–Nash equilibrium: $\pi_1^s > \pi_1^b$ and $\pi_2^s > \pi_2^b$.**
- **Follower earns higher profit than the leader: $\pi_1^s < \pi_2^s$.**
- **Being a follower is better.**

References

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