International Economics B
7. Monopolistic competition models

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What is “monopolistic competition”?

- **Chamberlin (1933) first introduced the idea of “monopolistic competition”**
  - **Chamberlin, E.H. (1933),** *The Theory of Monopolistic Competition*.
- **Defined as the following situation (Hart, 1985):**
  1. **There are many firms producing differentiated commodities;**
  2. **each firm is negligible, in the sense that it can ignore its impact on, and hence reactions from, other firms;**
  3. **free entry leads to zero profit of operating firms; but**
  4. **each firm faces a downward-sloping demand curve and hence equilibrium price exceeds marginal cost.**

Demand for differentiated goods

- **Types of product differentiation**
  - **Vertical differentiation: based on the quality of products**
  - **Horizontal differentiation: not based on the quality**
- **Approaches for demand theory that incorporates horizontal product differentiation:**
  1. **"Ideal variety" approach (Lancaster, 1975, 1979)**
     - **Consumers differ in their "ideal variety" of a differentiated good (a collection of different characteristics).**
  2. **"Love of variety" approach (Spence, 1976; Dixit and Stiglitz, 1977)**
     - **A representative consumer demanding many varieties of the differentiated good.**
  - **The two approaches lead to very similar results (Helpman and Krugman, 1985).**

Demand for differentiated goods: love of variety approach

- A representative consumer's utility function depends on the consumption of a differentiated good with N product varieties:

$$U = V(X), \quad V' > 0 \geq 0,$$

$$X \equiv \sum_{i=1}^{N} v(x_i), \quad v' > 0 > v''.$$

  - Utility from consuming the differentiated good is assumed symmetric over the product varieties.

- U **may depend on the consumption of a homogeneous numeraire good:** $U = \tilde{U}(x_0, V(X))$.
- **Functional forms for** $\tilde{U}(\cdot, \cdot)$
  - **Quasi-linear:** $\tilde{U}(x_0, V(X)) = V(X) + x_0$
  - **Cobb–Douglas:** $\tilde{U}(x_0, V(X)) = [V(X)]^{\alpha} x_0^{1-\alpha}$, $0 < \alpha < 1$

- Assume no homogeneous good and $V(X) = X$.
- The consumer maximizes $\sum_{i=1}^{N} v(x_i)$ subject to the budget constraint $\sum_{i=1}^{N} p_i x_i = I$.
  - I: consumer's income (assumed to be exogenous)
- FOC for optimal consumption of each variety:

$$v'(x_i) = \lambda p_i, \quad i = 1, \cdots, N$$

  - λ: Lagrange multiplier (marginal utility of income)
- Effect of a change in variety i's price on its demand when # of varieties is sufficiently large:

$$v'' dx_i = \lambda dp_i \quad \Rightarrow \quad \frac{dx_i}{dp_i} = \frac{\lambda}{v''} < 0$$

  - The impact of a price change on λ can be negligible.

- **Elasticity of demand for variety i:**

$$\epsilon_i = -\frac{dx_i}{dp_i}\frac{p_i}{x_i} = -\frac{v'}{x_i v''} > 0$$

- **A new product variety reduces the demand for each variety for a given price.**
  - **Since each variety is symmetric, $p_i x_i = I/N$**
    $\Rightarrow$ An increase in N reduces x_i for a given p_i.
  - **Increase in the variety $\Rightarrow$ downward shift in the demand curve for each variety**

Setup of the model

- A representative consumer derives utility over a differentiated good with N varieties.
  - Short run: N is exogenously given
  - Long run: N is endogenous because of entry/exit of firms
- Each firm producing the differentiated good has the same cost function $C(x_i) = cx_i + f$
  - $c > 0$: marginal cost
  - $f > 0$: fixed cost
- Each firm has a market power over the variety it produces.
- Economies of scale exist $\Rightarrow$ Each variety is produced by one firm.

Short-run equilibrium

- **Each firm behaves as a monopolist of the variety it produces.**
- **FOC:** $MR_i = MC_i$, **or**

$$p_i \left(1 - \frac{1}{\epsilon_i}\right) = c$$

- **Symmetry assumption** $\Rightarrow p_i = p \ \forall i$

Long-run equilibrium

- If $\pi_i > 0$ for a given # of varieties, new entry occurs, which makes a downward shift of the demand curve for each variety $\Rightarrow \pi_i \downarrow$
- In the long run, there is no more entry or exit; in equilibrium, $p = AC$, or

$$p = c + \frac{f}{x},$$

in addition to $MR = MC$.

Dixit–Stiglitz model

- Dixit and Stiglitz (1977) developed a tractable model of monopolistic competition
  - Applied in studies of international trade, economic growth, new economic geography, etc.
- Assume the following utility function:

$$U = \left(\sum_{i=1}^{N} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1.$$

  - CES (constant elasticity of substitution) function

- **Elasticity of substitution between variety i and variety j:**

$$\frac{d\ln(x_j/x_i)}{d\ln MRS_{ij}} = \frac{d\ln(x_j/x_i)}{d\ln(U_{x_i}/U_{x_j})}$$
$$= \frac{d\ln(x_j/x_i)}{d\ln[(x_j/x_i)^{1/\sigma}]} = \sigma$$

$\Rightarrow$ Elasticity of substitution between any two varieties is constant and equal to σ.

Demand for varieties

- Define the Lagrangian:

$$\mathcal{L} = \left(\sum_{i=1}^{N} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \lambda \left[I - \sum_{i=1}^{N} p_i x_i \right].$$

- FOC:

$$\frac{\partial \mathcal{L}}{\partial x_i} = \left(\sum_{i=1}^{N} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} x_i^{-\frac{1}{\sigma}} - \lambda p_i = 0, \quad i = 1, \cdots, N$$

- From the FOCs and the budget constraint, demand for each variety is derived as

$$x_i = \frac{p_i^{-\sigma} I}{\sum_{j=1}^{N} p_j^{1-\sigma}}.$$

- **Derivation of optimal x_i:**
  - **From the FOC,**

$$x_i = (\lambda p_i)^{-\sigma} \left(\sum x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

  - **Multiplying by p_i and summing over $i = 1, \cdots, N$,**

$$\sum p_i x_i = \lambda^{-\sigma} \left(\sum x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \sum p_i^{1-\sigma},$$

    the LHS of which is equal to, by budget constraint, I.

$$\Rightarrow \left(\sum x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \frac{\lambda^{\sigma} I}{\sum p_i^{1-\sigma}}$$

  - **Substituting the above into the 1st equation, the optimal x_i is derived.**

- **Price elasticity of demand faced by firm producing variety i:**

$$\epsilon_i = \sigma + \frac{p_i^{1-\sigma}}{\sum p_j^{1-\sigma}}(1-\sigma)$$

- **When # of varieties is large, the firm disregards the second term $\Rightarrow$ considers σ to be the elasticity of demand it faces.**
  - **If $p_j = p \; \forall i$ (which actually occurs in equilibrium), the 2nd term equals $(1-\sigma)/n$.**

Firm behavior

- As in the basic model, assume each firm has a cost function $C(x_i) = cx_i + f$, where $c, f > 0$.
- Each firm maximizes its profit $\pi_i = p_i x_i - C(x_i)$ subject to the demand function for the variety it produces, i.e., $x_i = p_i^{-\sigma} I / \sum p_j^{1-\sigma}$.
    - Although the firm determines the profit-maximizing price p_i as a monopolist, it does so taking I and $\sum p_j^{1-\sigma}$ as given; because there are many firms, each firm considers the household's expenditure and the economy's price index are independent of its actions.

- **Elasticity of demand is equal to $\sigma \Rightarrow$ FOC, i.e., $MR_i = MC_i$, can be rewritten as**

$$p_i \left(1 - \frac{1}{\sigma}\right) = c \quad \Rightarrow \quad p_i = \frac{\sigma}{\sigma - 1} c, \quad i = 1, \cdots, N.$$

- **RHS is common to all firms $\Rightarrow$ Same price for all varieties; $p_i = p \ \forall i$.**
- **Output of each variety:**

$$x_i = \frac{I}{N p_i} = \frac{(\sigma - 1) I}{N \sigma c}, \quad i = 1, \cdots, N.$$

Industry equilibrium

- Short run: # of firms (= varieties) is given.
  - Pair of price and output per variety:

$$p = \frac{\sigma}{\sigma - 1} c, \quad x = \frac{(\sigma - 1) I}{N \sigma c}$$

  - Each firm's profit:

$$\pi = (p - c)x - f = \frac{I}{N\sigma} - f$$

- If $\pi \neq 0$, entry/exit will occur.
  - If $\pi > 0$, potential entrants attempt to enter the industry so that they can enjoy positive profits as incumbent firms do.
  - If $\pi < 0$, some of the incumbent firm exit from the industry.

- **In the long-run equilibrium, the process of firm entry/exit ceases and thus $\pi = 0$.**
- **Equilibrium number of firms (and varieties) in the industry:**

$$\bar{N} = \frac{I}{\sigma f}$$

- **Equilibrium output of each variety:**

$$\bar{x} = \frac{(\sigma - 1)f}{c}$$

References

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