THEORY OF THE FIREBALL

by

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ABSTRACT

The successive stages of the fireball due to a nuclear explosion in air are defined (Sec. 2). This paper is chiefly concerned with Stage C, from the minimum in the apparent fireball temperature to the point where the fireball becomes transparent. In the first part of this stage (C I), the shock (which previously was opaque) becomes transparent due to decreasing pressure. The radiation comes from a region in which the temperature distribution is given essentially by the Taylor solution; the radiating layer is given by the condition that the mean free path is about 1/50 of the radius (Sec. 3). The radiating temperature during this stage increases about as $p^{-1/4}$, where *p* is the pressure.

To supply the energy for the radiation, a cooling wave proceeds from the outside into the hot interior (Sec. 5). When this wave reaches the isothermal sphere, the temperature is close t o its second maximum. Thereafter, the character of the solution changes; it is now dominated by the cooling wave (Stage C II). The temperature would decrease slowly (as $p^{1/6}$) if the problem were one-dimensional, but in fact it is probably nearly constant for the three-dimensional case (Sec. 6). The radiating surface shrinks slowly. The cooling wave eats into the isothermal sphere until this is completely used up. The inner past of the isothermal sphere, i.e., the part which has not yet been reached by the cooling wave, continues to expand adiabatically; it therefore cools very slowly and remains opaque.

After the entire isothermal sphere is used up, the fireball becomes transparent and the radiation drops rapidly. The ball will therefore be left at a rather high temperature (Sec. *7) ,* about 5000° .

The cooling wave reaches the isothermal sphere at a definite pressure $p_c \approx 5\left(\frac{\rho_1}{\rho_0}\right)^{1/3}$ bars, where ρ_1 is the ambient and ρ_0 the sea level density. The radiating temperature at this time is about

10000° . The slight dependence of physical properties on yield is exhibited in approximate formulae.

CONTENTS

1. INTRODUCTION

The radiation from the fireball has been studied intensively by many authors. Already in the Summer Study at Berkeley in 1942, Bethe and Teller[1] found that the energy transmitted by a nuclear explosion into air is immediately converted into x-rays, and studied the qualitative features of the transmission of these x-rays. At Los Alamos, Marsnak[2] and others showed that this radiation propagates as a wave, with a sharp front. Hirschfelder and Magee[3] gave the first comprehensive treatment of this early phase of the fireball development, and also studied some of the later phases, especially the role of NO_2.

May optical observations have been made in the numerous tests of atomic weapons. Some of tine results are contained in "Effects of Nuclear Weapons,"[4] pp. 70-84 (see also pp. 316-368) A summary of the spectroscopic observations up to 1956 was compiled by DeWitt[5]. Careful scrutiny of the extensive observational material would undoubtedly give a wealth of further information.

On the theoretical side, there has been some analytical and a good deal of numerical work. Analytical work has concentrated on the early phases. One of the most recent analytical papers on the early flow of radiation .(Stage A of Sec. 2) is by Freeman[6]. Brode and Gilmore[7] treat also Stage B, the radiation from the shock front, with particular emphasis on the dependence on altitude.

The most complete numerical calculation has been done by Brode[8] on a sea level megaton explosion. We shall use his results extensively, but for convenience we shall translate them to a yield of l megaton. Wherever the phenomena are purely hydrodynamic, we may simply scale the linear dimensions and the time by the cube root of the yield, and this is the principal use we shall make of Brode's results, e.g., in Sec. 3b. Where radiation is important, this scaling will give only a rough guide. Brode calculates pressures, temperatures, densities, etc., as functions of time and radius, for scaled (1-megaton) times of about 10^{-7} to 10 seconds. The calculations show clearly the stages in fireball and shock development, as defined in Sec. 2, at least Stages A t o C.

Brode and Meyerott[9] have considered the physical phenomena involved in the optical. "opening" of the shock after the minimum of radiation, Stage C I in the nomenclature of Sec. 2, especially the decrease in opacity due to decreased density and to the dissociation of NO_2.

Zel'dovich, Kompaneets and Raizer[10] have investigated how the energy for the radiation is supplied after the radiation minimum and have introduced the concept of a "cooling wave" moving into the hot fireball. The present report is largely concerned with an extension of the ideas of Zel'dovich et al. to the actual case of density varying with time, more general opacity function, radiation absorption varying with wave length, etc.

Much effort has been devoted to the calculation of fireballs at various altitudes. Brode[11] made such calculations in 1958, in connection with the test series of that year. Gilmore[12] made a prediction of the Bluegill explosion in 1962. Since then, many more refined calculations of Bluegill have been made.

For any understanding of fireball phenomena it is essential to have a good equation of state for air and good tables of absorption coefficients. For the equation of state, we have used Gilmore's tables[13], although Hilsenrath's[14] give more detail in some respects. The two calculations agree. Gilmore's equation was approximated analytically by Brode[8].

For the absorption coefficient we have used the tables by Meyerott et al.[15,16], which extend to 12,000°. These were supplemented by the work of Kivel and Bai1ey[17] and more recent work by Taylor and Kivel[18] on the free-free electron transitions in the field of neutral atoms and molecules. At higher temperatures, there are calculations by Gilmore and Latter[19], Karzas and Latter[20], and curves by Gilmore[21] which are brought to date periodically. The most recent calculation on the absorption of air at about 18,000° and above have been done by Stuart and Pyatt[22]. This temperature range is not of great importance for the problem of this paper, but is important for the expansion of the isothermal sphere inside the shock wave before it is reached by the cooling wave.

Brode[8] has used the average of the absorption coefficient over frequency, the opacity, which is sufficient for treating the internal flow of radiation. A realistic treatment of the flow t o the outside requires the absorption coefficient as a function of frequency; Brode merely wanted to obtain reasonable overall results for this flow. He approximated the opacity by an analytic expression. Also in most of the other work cited above an average opacity has been used. An exception is some of the recent work on the radiation flow in high altitude explosions (Bluegill), where the frequency dependence must be, and has been, taken into

account. Gilmore[21] has calculated and made available curves of effective opacity, in which the radiation mean free path was averaged (using a Rosseland weighting factor) only over those frequencies for which it is less than 1 kilometer.

This list of references on work on the fireball dynamics and opacity is far from complete.

2. PHASES IN DEVELOPMENT

The energy from a nuclear explosion is transmitted through the outer parts of the weapon, including its case, either by radiation (x-rays) or by shock or both. Whichever the mode of transmission inside the weapon, once the energy gets into the surrounding air, the energy will be transported by x-rays. This is because the air will be heated to such a high temperature (a million degrees or more) that transport by x-rays is much faster than by hydrodynamics. This stage of energy transport (Stage A) has been extensively studied by many authors (e.g., Hirschfelder and Magee in Report LA-2000) and will therefore not be further considered here.

During Stage A, temperatures are very high. The Planck spectrum of the air is in the x-ray or far ultraviolet region, and hence is immediately absorbed by the surrounding air. The very hot air is therefore surrounded by a cooler envelope, and only this envelope is visible to observers at a distance. The observable temperature therefore has little physical significance. It is observed that the size of the luminous sphere increases rapidly, and the total emission also increases, up to a first maximum.

When the temperature of the central sphere of air has fallen, by successive emission and re-absorption of x-rays, to about 300,000°, a hydrodynamic shock forms. The shock now moves faster than the temperature could propagate by radiation transport, The shock therefore separates from the very hot, nearly isothermal sphere at the center. This is Stage B in the development. The shock moves by simple hydrodynamics. Its front obeys the Hugoniot relations, the density being given by (3.4). Behind the front, the air expands adiabatically, and at a radius of 80% of the shock radius, .the density is apt to have fallen by a factor of 10 or more compared t o the shock density, while the temperature has risen by a comparable factor, (3.16). Thus the interior is at very low density, and hence the pressure must be nearly

constant (otherwise there would be very large accelerations which soon would equalize the pressure). This greatly simplifies the structure of the shock, and leads t o such simple relations as (3.2) between shock radius and pressure.

Well inside the shock, the "isothermal sphere" pursues its separate history. It continues t o engulf more material because radiation flow continues, though at a reduced rate. H. Brode has kindly calculated for me the temperature histories of several material points, based on his paper RM-2248. These histories clearly exhibit the expansion of the isothermal sphere in material coordinates. The expansion can also be treated by a semi-analytic method which I hope to discuss in a subsequent paper. The isothermal sphere remains isolated from the outside world until it is reached by the cooling wave, Sec. 5.

The radiation to the outside now comes from the shock. Early in Stage B, the shock has a precursor of lower temperature, caused by ultraviolet radiation from the shock, and the observable temperature is still below the shock temperature (Stage B I). However, the observable radius is very near the shock radius. Later, as the shock front cools down, the shock radiates directly and its temperature becomes directly observable (Stage B II). The first maximum in visible radiation probably occurs between Stages B I and B II. As the shock cools down, the radiation from the shock front decreases, and the observable temperature decreases t o a minimum (Ref. 4, par. 2.113, p. 75) of about 2000°.

When the shock is sufficiently cool, its front becomes transparent, and one can look into it to higher temperatures (Stage C). The central isothermal sphere, however, remains opaque and, for some time, invisible. Because higher temperatures are now revealed, the total radiation increases toward a .second maximum. This stage has been very little considered theoretically, except in numerical calculations, and forms the subject of this report.

We shall show that for some time in Stage C, the radiation comes from the air between isothermal sphere and shock front (Stage C I). During this time, the radiation can be calculated essentially from the temperature distribution which is set up by the adiabatic expansion of the material behind the shock (Secs. 3 and 5f, g). The temperature and total intensity of the radiation increase with time toward the larger, second maximum.

The energy for the radiation is largely supplied by a cooling wave (Sec. 5) which gradually eats into the hot interior. When this cooling wave reaches the isothermal sphere, the radiation temperature reaches its maximum (Sec. 5e); it then declines again as the cooling wave eats more deeply into the isothermal sphere (Stage C II). This process ends by the isothermal sphere being completely eaten away (Sec. 6).

After this has happened, the entire fireball, isothermal sphere and cooler envelope, is transparent to its own thermal radiation (Stage D). The molecular bands, which previously appeared in absorption, now appear in emission (Stage D I). Emission will lead to further cooling of the fireball, though more slowly than before. Soon, when the temperature falls below about 6000°, the emission becomes very weak, and subsequent cooling is almost entirely adiabatic (Stage D II). At sea level, the pressure may go down to 1 bar before the temperature falls below 6000°; in this case there is no Stage D II. At higher elevation, there usually is.

The fireball will then remain hot, at about 6000° or a somewhat lower temperature due to adiabatic expansion in Stage D II. The only process which can now lead to further cooling is the rise of the fireball, which leads t o further adiabatic expansion and, more important, to turbulent mixing at the surface with the ambient air (Stage E). The time required for this is typically 10 seconds or more, being determined by buoyancy.

At very high altitude, the shock wave never plays an important part, but radiation transport continues until the temperature gets too low for effective emission. In other words, Stage A continues to the end. Of course, a shock does form, but it is, so to speak, an afterthought, and it plays little part in the distribution of energy, At medium altitude, let us say, 10 to 30 kilometers, the stages are much the same as at sea level but the shock wave becomes transparent earlier, i.e., a t a higher temperature, because the density is lower; this means that the minimum emission comes earlier. Stage C proceeds similarly to sea level, but at the second maximum of radiation the pressure is still much above ambient, therefore Stage C II involves a greater radial expansion of the isothermal sphere than at sea level which proceeds simultaneously with the inward motion of the cooling wave. Moreover, there is much adiabatic expansion after the cooling wave has penetrated t the center. The temperature at which radiation stops is higher, due to the lower

density.

We believe that the theory developed in this paper will be useful in studying the dependence of phenomena on altitude (ambient pressure), but we have not yet exploited it for this purpose.

3. RADIATION FROM BEHIND THE SHOCK (Stage C I)

a., Role of NO_2

The diatomic species in equilibrium air, both neutrals and ions, show very little absorption in the visible at temperatures up to about 4000K. This is shown clearly in the tables by Meyerott et al.[16] We define "the visible" for the purposes of this paper, arbitrarily and incorrectly, as the frequency range

$$h\nu = 1/2 \text{ to } 2\text{-}3/4 \text{ [ev]}$$
$$4050 \text{ to } 22{,}300 \text{ [1/cm]}$$
$$2.48 \text{ to } 0.45 \text{ [}\mu\text{]} \qquad (3.1)$$

Then, even a t a density as high as $10\rho_0$ (ρ_0 = density of air at NPT = 1.29×10^{-3} [gm/cm^3]), the mean free path is never less than 100 meters at 4000°, 1000 meters at 3000°, and still longer at lower temperature. These values refer to $h\nu = 2-5/8$ [ev]; for lower frequencies, the mean free path is even longer.

In the sea-level shock wave from a megaton explosion, the temperature range from 3000 to 4000° occupies a distance of about 10 meters (see Sec. 3b). Thus this region is definitely transparent, even at the highest possible density of about $10\rho_0$. For explosions at higher altitude, this conclusion is even more true.

The tables by Meyerott et al. do not include absorption by tri-atomic (and more complicated) molecules, Of these, NO_2 is known to have strong absorption bands in the visible. After this paper was completed, I received new calculations by Gilmore[23] which include the effect of NO_2. The effect is very striking as is shown by Table I, which gives the absorption coefficient for the "typical" frequency $h\nu = 2-1/8$ [ev], and for several temperatures and densities (the absorption is strong from about 1-3/4 to 2-3/4 [ev]).

Because NO_2 is tri-atomic, its absorption depends strongly on density. As the shocked gas expands, the NO_2 dissociates and the gas becomes transparent. Brode and Meyerott[9] have calculated, under reasonable assumptions, the effect of this dissociation on the optical

properties of the fireball. We shall not discuss the effect of NO_2 any further, but shall assume that this substance has almost disappeared by the time we are considering.

b. Temperature Distribution behind the Shock

We wish to calculate the temperature distribution behind the shock. We can do this because the material which has gone through the shock expands very nearly adiabatically, as long as it is not engulfed by the internal, hot isothermal sphere. We are interested in the period when the shock temperature goes from about 4000 to a few hundred degrees, i.e., until the strong cooling wave (Sec. 5) starts. For a 1-megaton explosion, this corresponds roughly to t = 0.05 to 0.25 second.

At a given time, the pressure is nearly constant over most of the volume inside the shock, except for the immediate neighborhood of the shock; the shock pressure is roughly twice this constant, inside pressure. Comparing two material elements in the "inside" region, we may calculate their relative temperatures if we know their temperature when the shock traversed them, and assume adiabatic expansion from there on.

A material element which is initially at point r will be shocked when the shock radius is r. The shock pressure a t t h i s time is[1]

$$P_S(r) = 20(\gamma'_{AV} - 1)Yr^{-3} \tag{3.2}$$

where,

Y = yield in megatons

r = radius in kilometers,

$$\gamma' = 1 = \frac{P}{\rho E} = \frac{pressure}{energy\_per\_unit\_volume}, \tag{3.3}$$

and the average of γ' is taken over the volume inside the shock wave. The basis of (3.2) is that the total energy in the shocked volume, Y, is the volume times the average energy per unit volume, the latter is the average pressure divided by $\gamma' - 1$, and the average pressure is close to one-half of the shock pressure.

The density at the shock is

$$\rho_S = \frac{\gamma'_S + 1}{\gamma'_S - 1}\rho_0 \approx \frac{2}{\gamma'_S - 1}\rho_0, \tag{3.4}$$

where the subscript S refers to shock conditions. Now an examination of Gilmore's tables[13] shows that γ' does not change very much along an adiabat. As an example, we list in Table II certain quantities

[1] Notations for thermodynamic quantities similar t o those of Gilmore (Report

referring to some adiabats which will be particularly important for our theory. These are the adiabats for which the temperature T is between 4000 and 12,000° at a density of $0.1\rho_0$. In the second line we list the temperature T_S for the same entropy S at a density $\rho_S = 10\rho_0$. This is close enough to the shock density (3.4) so we may consider T_S as the temperature of the same material when the shock wave went through it. (Adiabatic expansion, i.e., no radiation transport, is assumed.) The third line gives $\gamma'-1$ for the "present" conditions, $\rho = 0.1\rho_0$ and T, the fourth line is the same quantity for the shock conditions. It is evident that $\gamma'-1$ is nearly constant for T = 4000 and 6000°, not so constant for 8000 and 12,000°. On the average $\gamma'-1 \approx 0.18$. The last two lines in Table II give the number of particles (atoms, ions, electrons) per original air molecule under "present" and shock conditions.

Assuming an adiabat of the constant $\gamma = \gamma'$, the density of a mass element is

$$\rho = \rho_S \left(\frac{P}{P_S} \right)^{1/\gamma} . \tag{3.5}$$

Now ρ_S is a constant, and at any given time, p is the same for all mass elements except those very close to the shock, hence

$$\rho \sim P_S^{-1/\gamma} \tag{3.6}$$

(~ means proportional to).

If we now introduce the abbreviation

$$m = r^3 , \tag{3.7}$$

which is proportional to the mass inside the mass element considered, and if we use (3.2), we find that at any given time

$$\rho \sim m^{1/\gamma} . \tag{3.8}$$

The radius R is given by

$$R^3 = \int \frac{dm}{\rho} \sim \int \frac{dm}{m^{1/\gamma}} = m \frac{\gamma - 1}{\gamma} - const . \tag{3.9}$$

We shall set the constant equal to zero which amounts to the (incorrect) assumption that (3.8) holds dam to m = 0. Actually, the isothermal sphere gives the constant a finite, positive value.

To find the temperature distribution, we note that the enthalpy

$$H = \frac{\gamma'}{\gamma'-1} \frac{p}{\rho} . \tag{3.10}$$

RM-1543).

We are using the enthalpy rather than the internal energy because the interior of the shock is at constant pressure, not constant density. The thermodynamic equation for H is

$$TdS = dH - Vdp \;. \tag{3.11}$$

At given pressure, i.e., given time, (3.8) to (3.10) give

$$H \sim \rho^{-1} \sim m^{-1/\gamma} \sim R^{-3/(\gamma-1)} \,. \tag{3.12}$$

Now the approximate relation between internal energy and temperature is given by Gilmore[13], Fig. 5, viz.,[2]

$$E = 4.2_5 \times 10^{11} \left(\frac{\rho}{\rho_0} \right)^{-0.1} T'^{1.5} \quad \text{[erg/gm]}, \tag{3.13}$$

where T' is the temperature in units of 10^4 degrees. Using $\rho = \dfrac{p}{E(\gamma'-1)}$ from (3.3) and setting $\gamma' = 1.18$, which is a reasonable average (see Table II) we may rewrite (3.13):

$$E = 7.0 \times 10^{11} p^{-1/9} T'^{5/3} \,, \tag{3.14}$$

where p is the pressure in bars. Since H is proportional to E, (3.12) and (3.14) give

$$T \sim H^{3/5} \sim R^{-1.8/(\gamma-1)} = R^{-\alpha} \,. \tag{3.15}$$

Using $\gamma'-1 = 0.18$, which is not far from the average of table II,

$$T \sim R^{-10} \,. \tag{3.16}$$

The numerical calculations of Brode[8] are in good agreement with this at the relevant times, from about 0.05 to 0.5 second.

It will be noted that (3.16) was obtained without integration of the hydrodynamic equations; it follows simply from the equation of state. The weakest assumptions are (1) the relation between E and T, (3.13), and (2) the neglect of the constant in (3.9). But in any case, T will be a very high power of R.

(to be continued)

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[2] The thermodynamic relation

$$\left(\frac{\partial E}{\partial V} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_V - p$$

leads to a relation between $\gamma - 1$ and the exponents in the relation

$$E = A_0 \rho^{-x} T^y$$

namely,

$$x = (\gamma - 1)(y - 1) \,.$$

Since we have chosen $\gamma = 1.18$ and y = 1.5 this relation gives x = 0.09. This is in sufficient agreement with x = 0.1 as used in (3.13).pp