

Harley Flanders
Differential Forms with Applications to the Physical Sciences
Dover, 1989 (1962)

Contents

FOREWORD

PREFACE TO THE DOVER EDITION

PREFACE TO THE FIRST EDITION

I. Introduction

- 1.1. Exterior Differential Forms
- 1.2. Comparison with Tensors

II. Exterior algebra

- 2.1. The Space of p-vectors
- 2.2. Determinants
- 2.3. Exterior Products
- 2.4. Linear Transformations
- 2.5. Inner Product Spaces
- 2.6. Inner Products of p-vectors
- 2.7. The Star Operator
- 2.8. Problems

III. The Exterior Derivative

- 3.1. Differential Forms
- 3.2. Exterior Derivative
- 3.3. Mappings
- 3.4. Change of Coordinates
- 3.5. An Example from Mechanics
- 3.6. Converse of the Poincare Lemma
- 3.7. An Example
- 3.8. Further Remarks
- 3.9. Problems

IV. Applications

- 4.1. Moving Frames in E^3
- 4.2. Relation between Orthogonal and Skew-symmetric Matrices
- 4.3. The 6-dimensional Frame Space
- 4.4. The Laplacian, Orthogonal Coordinates
- 4.5. Surfaces
- 4.6. Maxwell's Field Equations
- 4.7. Problems

V. Manifolds and Integration

- 5.1. Introduction

- 5.2. Manifolds
- 5.3. Tangent Vectors
- 5.4. Differential Forms
- 5.5. Euclidean Simplices
- 5.6. Chains and Boundaries
- 5.7. Integration of Forms
- 5.8. Stokes' Theorem
- 5.9. Periods and De Rham's Theorems
- 5.10. Surfaces; Some Examples
- 5.11. Mappings of Chains
- 5.12. Problems

VI. Applications in Euclidean space

- 6.1. Volumes in E^n
- 6.2. Winding Numbers, Degree of a Mapping
- 6.3. The Hopf Invariant
- 6.4. Linking Numbers, the Gauss Integral, Ampère's Law

VII. Applications to Differential Equations

- 7.1. Potential Theory
- 7.2. The Heat Equation
- 7.3. The Frobenius Integration Theorem
- 7.4. Applications of the Frobenius Theorem
- 7.5. Systems of Ordinary Equations
- 7.6. The Third Lie Theorem

VIII. Applications to Differential Geometry

- 8.1. Surfaces (Continued)
- 8.2. Hypersurfaces
- 8.3. Riemannian Geometry, Local Theory
- 8.4. Riemannian Geometry, Harmonic Integrals
- 8.5. Affine Connection
- 8.6. Problems

IX. Applications to Group Theory

- 9.1. Lie Groups
- 9.2. Examples of Lie Groups
- 9.3. Matrix Groups
- 9.4. Examples of Matrix Groups
- 9.5. Bi-invariant Forms
- 9.6. Problems

X. Application to Physics

- 10.1. Phase and State Space

10.2. Hamiltonian Systems

10.3. Integral-invariants

10.4. Brackets

10.5. Contact Transformations

10.6. Fluid Mechanics

10.7. Problems

BIBLIOGRAPHY

GLOSSARY OF NOTATION

INDEX

GLOSSARY OF NOTATION

A. Spaces

- E^n Euclidean n -space.
- R The set of real numbers, also considered as E^1 , the Euclidean line.
- U, V, \dots Open sets (in E^n , or on a manifold).
- L, M, \dots Vector spaces.
- $\wedge^p L$ The space of p -vectors on L .
- M, N, \dots Manifolds.
- $F^p(U)$ The collection of all p -forms on U .
- Cartesian product. If S and T are arbitrary sets (collections of objects), their *cartesian product* is the set
- $$S \times T$$
- consisting of all ordered pairs (s, t) where s belongs to S and t to T .
- $\times^2 S$ This is the cartesian product $S \times S$ of S with itself. Similarly $\times^3 S = S \times S \times S$, etc.
- $S \cap T$ This is the intersection of the sets S and T . For example, if $S = \{1, 2, 5, 7\}$ and $T = \{2, 3, 7, 9\}$, then $S \cap T = \{2, 7\}$.
- I The unit interval $0 \leq t \leq 1$.

B. Functions

Mapping. A mapping is a smooth function ϕ from one space M to another N . We write

$$\phi: M \rightarrow N.$$

Composite mapping. If $\phi: M \rightarrow N$ and $\psi: N \rightarrow P$, then we may form the composite mapping $\psi \circ \phi: M \rightarrow P$. It is defined by

$$(\psi \circ \phi)(x) = \psi[\phi(x)] \quad \text{for } x \text{ in } M.$$

Linear functional. A linear transformation on a linear space L to the one-dimensional space R of real numbers.

Jacobian. If $u^i = u^i(x^1, \dots, x^n)$ ($i = 1, \dots, n$), the Jacobian of this mapping is the determinant

$$\left| \partial u^i / \partial x^j \right|.$$

- ϕ^* The mapping on differential forms induced by the mapping ϕ between spaces.
- ϕ_* The mapping on chains induced by the mapping ϕ between spaces.

C. Special symbols

tA The transpose of the matrix A , obtained from A by interchanging rows and columns.

$|A|$ The determinant of the linear transformation (matrix) A .

$\dim L$ The dimension of the linear space L .

σ^H Here $H = \{h_1, \dots, h_p\}$, a set of indices in increasing order,

$$1 \leq h_1 < \dots, h_p \leq n \text{ and } \sigma^H = \sigma^{h_1} \wedge \dots \wedge \sigma^{h_p}.$$

H' This is the complementary set of indices. For example, if $n = 8$ and $H = \{2, 3, 5, 6\}$, then $H' = \{1, 4, 7, 8\}$.

$\operatorname{sgn} \pi$ If π is a permutation on $\{1, 2, \dots, n\}$, then $\operatorname{sgn} \pi = 1$ if π is effected by an even number of interchanges (of two numbers) and $\operatorname{sgn} \pi = -1$ if π is effected by an odd number of interchanges.

$*$ The star operator.

(α, β) The inner product.

∂ The boundary operator.

$dx^1 \wedge \dots \wedge \overline{dx^i} \wedge \dots \wedge dx^n$ means $dx^1 \wedge \dots \wedge dx^{i-1} \wedge dx^{i+1} \wedge \dots \wedge dx^n$.

(The circumflex indicates omission.)