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Differential Forms with Applications to the Physical Sciences

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Contents

GLOSSARY OF NOTATION

A. Spaces

E^n Euclidean n-space.

R The set of real numbers, also considered as E^1, the Euclidean line.

$U, V, \ldots$ Open sets (in E^n, or on a manifold).

$L, M, \ldots$ Vector spaces.

$\wedge^p L$ The space of p-vectors on L.

$M, N, \ldots$ Manifolds.

$F^p(U)$ The collection of all p-forms on U.

Cartesian product. If S and T are arbitrary sets (collections of objects), their *cartesian product* is the set

$$S \times T$$

consisting of all ordered pairs (s, t) where s belongs to S and t to T.

$\times^2 S$ This is the cartesian product $S \times S$ of S with itself. Similarly $\times^3 S = S \times S \times S$, etc.

$S \cap T$ This is the intersection of the sets S and T. For example, if S = {1, 2, 5, 7} and T = {2, 3, 7, 9}, then $S \cap T$ = {2, 7}.

I The unit interval $0 \le t \le 1$.

B. Functions

Mapping. A mapping is a smooth function ϕ from one space M to another N. We write

$$\phi : M \to N .$$

Composite mapping. If $\phi : M \to N$ and $\psi : N \to P$, then we may form the composite mapping $\psi \circ \phi : M \to P$. It is defined by

$$(\psi \circ \phi)(x) = \psi[\phi(x)] \quad \text{for } x \text{ in } M.$$

Linear functional. A linear transformation on a linear space L to the one- dimensional space R of real numbers.

Jacobian. If $u^i = u^i(x^1, \ldots, x^n)$ (i = 1,…,n), the Jacobian of this mapping is the determinant

$$\left| \partial u^i / \partial x^j \right| .$$

ϕ^* The mapping on differential forms induced by the mapping ϕ between spaces.

ϕ_* The mapping on chains induced by the mapping ϕ between spaces.

C. Special symbols

${}^{t}A$	The transpose of the matrix A, obtained from A by interchanging rows and columns.
$\lvert A \rvert$	The determinant of the linear transformation (matrix) A.
$\dim L$	The dimension of the linear space L.
σ^H	Here $H = \{h_1, \ldots, h_p\}$, a set of indices in increasing order, $1 \le h_1 < \cdots, h_p \le n$ and $\sigma^H = \sigma^{h_1} \wedge \cdots \wedge \sigma^{h_p}$.
H'	This is the complementary set of indices. For example, if $n = 8$ and $H = \{2, 3, 5, 6\}$, then $H' = \{1, 4, 7, 8\}$.
$\operatorname{sgn} \pi$	If π is a permutation on $\{1, 2, \ldots, n\}$, then $\operatorname{sgn} \pi = 1$ if π is effected by an even number of interchanges (of two numbers) and $\operatorname{sgn} \pi = -1$ if π is effected by an odd number of interchanges.
*	The star operator.
(α, β)	The inner product.
∂	The boundary operator.

$dx^1 \wedge \cdots \wedge \overline{dx^i} \wedge \cdots \wedge dx^n$ means $dx^1 \wedge \cdots \wedge dx^{i-1} \wedge dx^{i+1} \wedge \cdots \wedge dx^n$. (The circumflex indicates omission.)