CHAPTER XI
THE INTERSECTION OF SURFACES OF DISCONTINUITY

§109. Rarefaction waves

The line of intersection of two shock waves is, mathematically, a singular line of two functions describing the gas flow. The vertex of an acute angle on the surface of a body past which the gas flows is always such a singular line. It is found that the gas flow near the singular line can be investigated in a general manner (L. Prandtl and T. Meyer 1908).

In considering the region near a small segment of the singular line, we may regard the latter as a straight line, which we take as the *z*-axis in a system of cylindrical polar coordinates r, ϕ, z. Near the singular line, all quantities depend considerably on the angle ϕ, but their dependence on the coordinate *r* is only slight, and for sufficiently small *r* it can be neglected. The dependence on the coordinate *z* is also unimportant; the change in the flow pattern over a small segment of the singular line may be neglected.

Thus we have to investigate a steady flow in which all quantities are functions of ϕ only. The equation of conservation of entropy, $\boldsymbol{v}\cdot\boldsymbol{grad}s = 0$, gives $v_\phi \dfrac{ds}{d\phi} = 0$, whence s = constant,[1] i.e., the flow is isentropic. In Euler's equation we can therefore replace $\dfrac{\boldsymbol{grad}p}{\rho}$ by $\boldsymbol{grad}w$: $(\boldsymbol{v}\cdot\boldsymbol{grad})\boldsymbol{v} = -\boldsymbol{grad}w$. In cylindrical polar coordinates, we have three equations:

$$\begin{cases} \dfrac{v_\phi}{r}\dfrac{dv_r}{d\phi} - \dfrac{v_\phi^{\,2}}{r} = 0 \\ \dfrac{v_\phi}{r}\dfrac{dv_\phi}{d\phi} + \dfrac{v_r v_\phi}{r} = -\dfrac{1}{r}\dfrac{dw}{d\phi} \\ v_\phi \dfrac{dv_z}{d\phi} = 0 \end{cases}.$$

From the last of these we have $v_z = \text{constant}$, and without loss of generality we can put $v_z = 0$, regarding the flow as two-dimensional; this is simply a matter of suitably defining the velocity of the coordinate system along the *z*-axis. The first two equations can be written

$$v_\phi = \frac{dv_r}{d\phi}, \tag{109.1}$$

$$v_\phi\left(\frac{dv_\phi}{d\phi} + v_r\right) = -\frac{1}{\rho}\frac{dp}{d\phi} = -\frac{dw}{d\phi}. \tag{109.2}$$

Substituting (109.1) in (109.2), we have

$$v_\phi \frac{dv_\phi}{d\phi} + v_r \frac{dv_r}{d\phi} = -\frac{dw}{d\phi},$$

or, integrating,

$$w + \frac{1}{2}(v_\phi^{\,2} + v_r^{\,2}) = \text{constant}. \tag{109.3}$$

We may notice that equation (109.1) implies that **curl** $\boldsymbol{v}$ = 0, i.e., we have potential flow, as a result of which Bernoulli's equation (109.3) holds.

Next, the equation of continuity, $div(\rho\boldsymbol{v}) = 0$, gives

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[1] If $v_\phi = 0$, we easily deduce from the equations of motion given below that $v_r = 0$, $v_z \neq 0$. Such a flow would correspond to the intersection of surfaces of tangential discontinuity (with a discontinuous velocity v_z), and is of no interest, since such discontinuities are unstable.

$$\rho v_r + \frac{d}{d\phi}(\rho v_\phi) = \rho\left(v_r + \frac{dv_\phi}{d\phi}\right) + v_\phi \frac{d\rho}{d\phi} = 0 . \tag{109.4}$$

Using (109.2), we obtain

$$\left(\frac{dv_\phi}{d\phi} + v_r\right)\left(1 - v_\phi{}^2 \frac{d\rho}{dp}\right) = 0 .$$

The derivative $\frac{dp}{d\rho}$, or more correctly $\left(\frac{dp}{d\rho}\right)_s$, is just the square of the velocity of sound. Thus

$$\left(\frac{dv_\phi}{d\phi} + v_r\right)\left(1 - \frac{v_\phi{}^2}{c^2}\right) = 0 . \tag{109.5}$$

This equation can be satisfied in either of two ways. Firstly, we may have $\frac{dv_\phi}{d\phi} + v_r = 0$. Then, from (109.2), p = constant and ρ = constant, and from (109.3) we find that $v^2 = v_r{}^2 + v_\phi{}^2$ = constant, i.e., the velocity is constant in magnitude. It is easy to see that in this case the velocity is constant in direction also. The angle χ between the velocity and some given direction in the plane of the motion is (Fig. 96)

$$\chi = \phi + \arctan\frac{v_\phi}{v_r} . \tag{109.6}$$

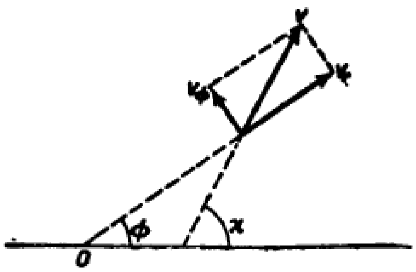

FIG. 96

Differentiating this expression with respect to ϕ and using formulae (109.1) and (109.2), we easily obtain

$$\frac{d\chi}{d\phi} = -\frac{v_t}{\rho v_\phi v^2}\frac{dp}{d\phi} . \tag{109.7}$$

Since p = constant, it follows that χ = constant. Thus, if the first factor in (109.5) is zero, we have the trivial solution of a uniform flow.

Secondly, equation (109.5) can be satisfied by putting $1 - \frac{v_\phi{}^2}{c^2} = 0$, i.e., $v_\phi = \pm c$. The radial velocity is given by (109.3). Denoting the constant in that equation by w_0, we find that

$$\begin{cases} v_\phi = \pm c \\ v_r = \pm\sqrt{2(w_0 - w) - c^2} \end{cases} .$$

In this solution, the velocity component v_ϕ perpendicular to the radius vector is equal to the local velocity of sound at every point. The total velocity $v = \sqrt{v_\phi{}^2 + v_r{}^2}$ therefore exceeds that of sound. Both the magnitude and the direction of the velocity are different at different

points. Since the velocity of sound cannot vanish, it is clear that the function $v_\phi(\phi)$, which is continuous, must everywhere be + *c*, or else everywhere - *c*. By measuring the angle ϕ in the appropriate direction, we can take $v_\phi = c$. We shall see below that the choice of the sign of v_r follows from physical considerations, and that the plus sign must be taken. Thus

$$\begin{cases} v_\phi = c \\ v_r = \sqrt{2(w_0 - w) - c^2} \end{cases}. \qquad (109.8)$$

From the equation of continuity (109.4) we have $d\phi = -d(\rho v_\phi)/\rho v_r$. Substituting (109.8) and integrating, we have

$$\phi = -\int \frac{d(\rho c)}{\rho\sqrt{2(w_0 - w) - c^2}}. \qquad (109.9)$$

If the equation of state of the gas and the adiabatic equation are known (we recall that *s* is constant), this formula can be used to determine all quantities as functions of the angle ϕ. Thus formulae (109.8) and (109.9) completely determine the gas flow.

Let us now study in more detail the solution which we have obtained. First of all, we notice that the straight lines ϕ = constant intersect the streamlines at every point at the Mach angle (whose sine is $\frac{v_\phi}{v} = \frac{c}{v}$), i.e., they are **characteristics**. Thus one family of characteristics (in the *xy*-plane) is a **pencil** of straight lines through the singular point, and has an important property in this case: all quantities are constant along each characteristic. In this respect the solution concerned plays the same part in the theory of steady two-dimensional flow as does the similarity flow discussed in §99 in the theory of non-steady one-dimensional flow. We shall return to this point in §115.

It is seen from (109.9) that $(\rho c)' < 0$, the prime denoting differentiation with respect to ϕ. Putting $(\rho c)' = \rho' \frac{d(\rho c)}{d\rho}$ and noticing that the derivative $\frac{d(\rho c)}{d\rho}$ is positive (see (99.9)), we find that $\rho' < 0$, and therefore so are the derivatives $p' = c^2\rho'$ and $w' = p'/\rho$. Next, from the fact that *w*' is negative it follows that the velocity $v = \sqrt{2(w_0 - w)}$ increases with ϕ. Finally, from (109.7), $\chi' > 0$. Thus we have

$$\frac{dp}{d\phi} < 0, \quad \frac{d\rho}{d\phi} < 0, \quad \frac{dv}{d\phi} > 0, \quad \frac{d\chi}{d\phi} > 0. \qquad (109.10)$$

In other words, when we go round the singular point in the direction of flow, the density and pressure decrease, while the magnitude of the velocity increases and its direction rotates in the direction of flow.

The flow just described is often called a ***rarefaction wave***, and we shall use this name in what follows.

It is easy to see that a rarefaction wave cannot exist throughout the region surrounding the singular line. For, since *v* increases monotonically with ϕ, a complete circuit round the origin (i.e., a change of ϕ by 2π) would give a value for *v* different from the initial one, which is impossible. For this reason, the actual pattern of flow round the singular line must be composed of a series of sectors separated by planes ϕ = constant which are surfaces of discontinuity. In each of these regions we have either a **rarefaction wave** or a **flow with constant velocity.** The number and nature of these regions for various particular cases will be established in the following sections. Here we shall simply mention that the boundary between a rarefaction wave and a uniform flow must be a **weak discontinuity**: it cannot be a tangential discontinuity (of v_r), since the normal velocity component $v_\phi = c$ does not vanish on it.

Nor can it be a shock wave, since the normal velocity component v_ϕ must be greater than the velocity of sound on one side of such a discontinuity and smaller on the other side, whereas in our problem we always have $v_\phi = c$ on one side of the boundary.

An important conclusion can be drawn from the foregoing. Disturbances which cause weak discontinuities leave the singular line (the *z*-axis) and are propagated away from it. This means that the weak discontinuities bounding the rarefaction wave must be ones which leave this line, i.e., the velocity component v_r tangential to the weak discontinuity must be positive. This justifies the choice of the sign of v_r made in (109.8).

Let us now apply these formulae to a polytropic gas. In such a gas $w = \frac{c^2}{\gamma - 1}$, while the equation of the Poisson adiabatic can be written

$$\rho c^{-1/(\gamma-1)} = \text{constant}, \quad p c^{-2\gamma/(\gamma-1)} = \text{constant}; \tag{109.11}$$

cf. (99.13). Using these formulae, we can put the integral (109.9) in the form

$$\phi = -\sqrt{\frac{\gamma+1}{\gamma-1}} \int \frac{dc}{\sqrt{c_*^2 - c^2}},$$

where c_* is the critical velocity (see (83.14)). Hence

$$\phi = -\sqrt{\frac{\gamma+1}{\gamma-1}} \arccos \frac{c}{c_*} + \text{constant},$$

or, if we measure ϕ in such a way that the constant is zero,

$$v_\phi = c = c_* \cos \sqrt{\frac{\gamma-1}{\gamma+1}} \phi. \tag{109.12}$$

According to formula (109.8) we therefore have

$$v_r = \sqrt{\frac{\gamma+1}{\gamma-1}} c_* \sin \sqrt{\frac{\gamma-1}{\gamma+1}} \phi. \tag{109.13}$$

Next, using the Poisson adiabatic equation in the form (109.11), we can find the pressure as a function of the angle ϕ:

$$p = p_* \cos^{2\gamma/(\gamma-1)} \sqrt{\frac{\gamma-1}{\gamma+1}} \phi. \tag{109.14}$$

Finally, we have for the angle χ (109.6)

$$\chi = \phi + \arctan\left(\sqrt{\frac{\gamma-1}{\gamma+1}} \cot \sqrt{\frac{\gamma-1}{\gamma+1}} \phi \right), \tag{109.15}$$

the angles χ and ϕ being measured from the same initial line.

Since we must have $v_r > 0$, $c > 0$, the angle ϕ in these formulae can vary only between 0 and $\phi_{\max}$, where

$$\phi_{\max} = \frac{1}{2}\pi \sqrt{\frac{\gamma+1}{\gamma-1}}. \tag{109.16}$$

This means that the rarefaction wave can occupy a sector whose angle does not exceed $\phi_{\max}$; for a diatomic gas (air, for example), this angle is 219.3°. When ϕ varies from 0 to $\phi_{\max}$, the angle χ varies from $\pi/2$ to $\phi_{\max}$. Thus the direction of the velocity in the rarefaction wave can turn through an angle not exceeding $\phi_{\max} - \frac{\pi}{2}$ (= 129.3° for air).

For $\phi = \phi_{\max}$ the pressure is zero. In other words, if the rarefaction wave occupies the

maximum angle, the weak discontinuity on one side is a boundary with a vacuum, and is, of course, a streamline; we have $v_\phi = c = 0$, $v_r = \sqrt{\frac{\gamma+1}{\gamma-1}} c_* = v_{\max}$, i.e., the velocity is radial and attains its limiting value $v_{\max}$ (see §83).

Figure 97 shows graphs of p/p_*, c_*/v and $\chi_1 = \chi - \pi/2$ as functions of the angle ϕ for air ($\gamma = 1.4$).

It is useful to note the form of the curve in the $v_x v_y$-plane defined by formulae (109.12) and (109.13) (called the ***velocity hodograph***). It is an arc of an **epicycloid** between circles with radii $v = c_*$ and $v = v_{\max}$ (Fig. 98).

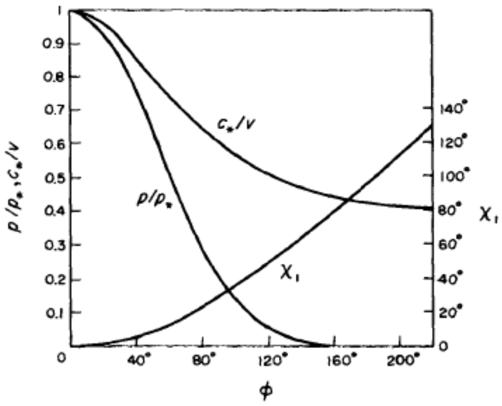

FIG. 97

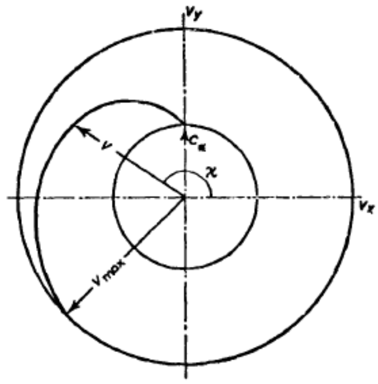

FIG. 98

PROBLEMS

Problem 1. Determine the form of the streamlines in a rarefaction wave.

Solution. The equation of the streamlines for two-dimensional flow is, in polar coordinates, $\frac{dr}{v_r} = r\frac{d\phi}{v_\phi}$. Substituting (109.12) and (109.13) and integrating, we obtain

$$r = r_0 \cos^{-(\gamma+1)/(\gamma-1)} \sqrt{\frac{\gamma-1}{\gamma+1}}\phi .$$

These streamlines form a family of similar curves concave toward the origin, which is the centre of similarity.

Problem 2. Determine the maximum possible angle between the weak discontinuities bounding a rarefaction wave, for given values v_1, c_1 of the gas velocity and the velocity of sound at one discontinuity.

Solution. The angle ϕ corresponding to the first discontinuity is, by (109.12),

$$\phi_1 = \sqrt{\frac{\gamma+1}{\gamma-1}} \arccos\frac{c_1}{c_*} .$$

The value of ϕ_2 is $\phi_{\max}$, so that the angle required is

$$\phi_2 - \phi_1 = \sqrt{\frac{\gamma+1}{\gamma-1}} \arcsin\frac{c_1}{c_*} .$$

The critical velocity c_* is given in terms of v_1 and c_1 by Bernoulli's equation:

$$w_1 + \frac{1}{2}v_1^2 = \frac{c_1^2}{\gamma - 1} + \frac{1}{2}v_1^2 = \frac{\gamma + 1}{2(\gamma - 1)}c_*^2 .$$

The maximum possible angle through which the gas velocity can turn in a rarefaction wave is accordingly, by (109.15), the difference $\chi_{\max} = \chi(\phi_1) - \chi(\phi_2)$:

$$\chi_{\max} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \arcsin\frac{c_1}{c_*} - \arcsin\frac{c_1}{v_1} .$$

As a function of v_1 / c_1, $\chi_{\max}$ is greatest for $v_1 / c_1 = 1$;

$$\chi_{\max} = \frac{1}{2}\pi\left(\sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1\right).$$

For $v_1 / c_1 \to \infty$, $\chi_{\max}$ tends to zero:

$$\chi_{\max} = \frac{2}{\gamma - 1}\frac{c_1}{v_1} .$$

§110. Classification of intersections of surfaces of discontinuity

Shock waves can intersect along a line. In considering the flow near a small segment of this line, we can assume that it is a straight line, and that the surfaces of discontinuity are planes. It is therefore sufficient to discuss the intersection of plane shock waves.

The line of intersection of two discontinuities is, mathematically, a singular line, as has already been mentioned at the beginning of §109. The flow pattern near this line consists of a number of sectors, in each of which we have either uniform flow or a rarefaction wave of the kind described in §109. It is possible to give a general classification of the possible types of intersection of surfaces of discontinuity.[2]

First of all, we must make the following remark. If the gas flow on both sides of a shock wave is supersonic, then (as mentioned at the beginning of §92) we can speak of the "direction" of the shock wave, and accordingly distinguish shock waves leaving the line of intersection from those reaching it. In the former case, the tangential velocity component is directed away from the line of intersection, and we can say that the disturbances which cause the discontinuity leave this line. In the latter case, the disturbances leave a point not on the line of intersection.

If the flow on one side of the shock wave is subsonic, then disturbances are propagated in both directions along its surface, and the "direction" of the shock has, strictly, no meaning. In the arguments given below, however, what is important is that disturbances leaving the point of intersection can be propagated along such a discontinuity. In this sense, such shock waves play the same part in the following discussion as the purely supersonic shocks which leave the intersection, and we shall include both kinds in the term "shocks which leave the intersection".

Figures 99-104 show the flow patterns in a plane perpendicular to the line of intersection. We can assume, without loss of generality, that the flow occurs in this plane. The velocity component parallel to the line of intersection (which lies in all the planes of discontinuity)

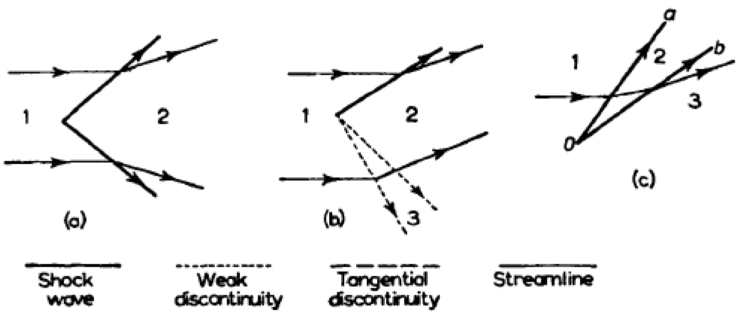

FIG. 99

[2] This is due to L. D. Landau (1944), with supplementary points (relating to the interaction of shock waves with

must be the same in all regions round the line of intersection, and can therefore be made to vanish by an appropriate choice of the coordinate system.

Let us first mention some configurations that are certainly impossible. It is easy to see that there can be no intersection of shock waves in which no shock reaches the intersection. For instance, in the intersection of two shock waves leaving the intersection, shown in Fig. 99a, the streamlines of the flow incident from the left would deviate in opposite directions, whereas the velocity should be constant throughout region 2, and this difficulty cannot be overcome by adding any further discontinuities in region 2.[3] Similarly, we can see that the intersection of a shock wave and a rarefaction wave both leaving the intersection, shown in Fig. 99b, is impossible; although the velocity in region 2 can be constant in direction, the pressure cannot be constant, since it increases in a shock wave but decreases in a rarefaction wave.

Next, since the intersection cannot affect shock waves reaching it, the simultaneous intersection (along a common line) of more than two such waves, which are due to other causes, would be an improbable coincidence. Thus only one or two shock waves can reach the intersection.

The following fact is very important. The gas flowing past a point of intersection can pass through only one shock or rarefaction wave leaving this point. For example, let the gas pass through two successive shock waves leaving the point *O*, as shown in Fig. 99c. Since the normal velocity component v_{2n} behind the shock *Oa* is $v_{2n} < c_2$, the velocity component in region 2 normal to the shock *Ob* must also be less than c_2, in contradiction to a fundamental property of shock waves. Similarly, we can see that the gas cannot pass through two successive rarefaction waves, or a shock wave and a rarefaction wave, leaving the point *O*.

These arguments evidently cannot be extended to shock waves reaching the point of intersection.

We can now proceed to enumerate the possible types of intersection. Figure 100 shows an intersection involving one shock wave *Oa* reaching it and two shock waves *Ob*, *Oc* leaving it. This case may be regarded as the splitting of one shock wave into two.[4] It is easy to see that, besides the two shock waves leaving, there must be formed a tangential discontinuity *Od* lying between them, which separates the gas flowing through *Ob* from that flowing through *Oc*.[5] For the shock *Oa* is due to other causes, and is therefore completely defined. This means that the thermodynamic quantities (p and ρ, say) and the velocity $\boldsymbol{v}$ have given values in regions 1 and 2. There remain at our disposal, therefore, only two quantities (the angles giving the directions of the discontinuities *Ob* and *Oc*) with which to satisfy, in general, four conditions (the constancy of p, ρ and two velocity components) in the region 3-4, which would have to be satisfied in the absence of the tangential discontinuity *Od*. The addition of the latter,

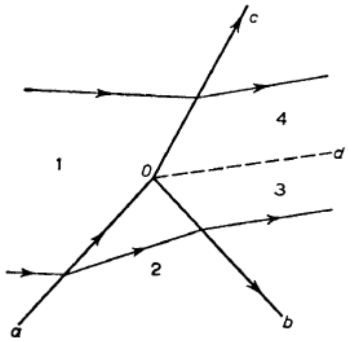

FIG. 100

tangential and weak discontinuities) added by S. P. D'yakov (1954).

[3] In order not to encumber the discussion with repetitive arguments, we shall not give similar considerations for cases where there are regions of subsonic flow and the shock leaving the intersection is actually a shock wave bounded by a subsonic region.

[4] It should be noticed that a shock wave cannot divide into a shock and a rarefaction wave; it is easily seen that the changes in the pressure and the direction of the velocities in the two waves leaving cannot be reconciled.

[5] As usual, the tangential discontinuity in reality becomes a turbulent region.

however, reduces the number of conditions to two (the constancy of the pressure and of the direction of the velocity).

An arbitrary shock wave, however, cannot divide in this manner. A shock wave reaching the intersection is defined by two parameters (for a given thermodynamic state of gas 1), say the Mach number M_1 of the incident stream and the ratio of pressures p_1 / p_2. It can divide in two only in a certain region in the plane of these two parameters.[6]

Intersections involving two shock waves reaching them can be regarded as "**collisions**" of two shocks due to other causes. Here two essentially different cases are possible, as shown in Fig. 101.

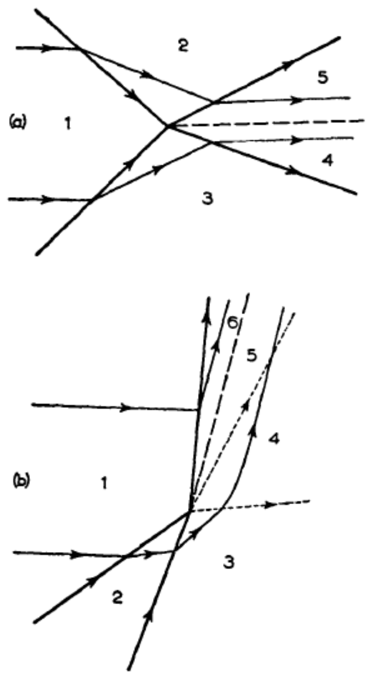

Fig. 101

In the first case, the collision of two shock waves results in two other shock waves leaving the point of intersection. If all the necessary conditions are to be fulfilled, a tangential discontinuity must again be formed, and it must lie between the two resulting shock waves.

In the second case, instead of two shock waves, there are formed one shock wave and one rarefaction wave.

Two colliding shock waves are defined by three parameters (for instance, M_1 and the ratios p_1 / p_2, p_1 / p_3). The types of intersection just described are possible only for certain ranges of values of these parameters. If the values of the parameters do not lie in these ranges, the collision of the shock waves must be preceded by their breaking up.

Let us next consider the types of intersection that can occur when a shock wave meets a tangential discontinuity.

Figure 102a shows the reflection of a shock wave from the boundary between gas in motion and gas at rest. Region 5 contains gas at rest, separated from the gas in motion by a tangential discontinuity. In the two regions 1 and 4 adjoining it, the pressure must be the same and equal to p_5. Since the pressure increases in a shock wave, it is clear that the shock must be reflected from the tangential discontinuity as a rarefaction wave 3, which reduces the

[6] The determination of this region involves very laborious algebraic or numerical calculations. Again, the "direction" of the shock waves is important. Cases with two shocks reaching the intersection and one leaving it would constitute an intersection of two discontinuities due to other causes, and therefore reaching the point of intersection with given values of all parameters. Their fusion into one shock is possible only when these arbitrary parameters are related in a certain way, and this would be an improbable coincidence.

pressure to its initial value. The tangential discontinuity has a **kink** at the point of intersection.

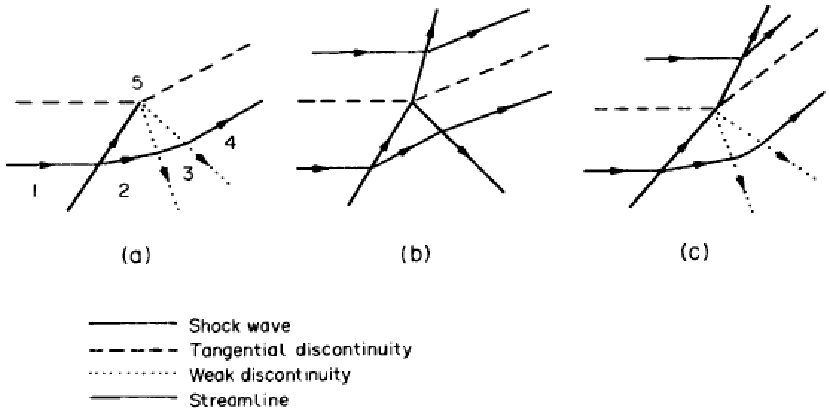

FIG. 102

The intersection of a shock wave with a tangential discontinuity having a non-zero but subsonic flow velocity on the other side is impossible. Neither a shock wave nor a rarefaction wave can penetrate into a subsonic region, and in this region there can thus be only a trivial flow with constant velocity, so that the tangential discontinuity cannot have a kink. The shock wave cannot be reflected as a rarefaction wave, since this would necessarily give the tangential discontinuity a kink; reflection as a shock wave is also impossible, since it would then be impossible to satisfy the condition of equal pressures at the tangential discontinuity.

If the flow on either side of a tangential discontinuity is supersonic, there are two possible configurations. In one (Fig. 102b), in addition to the incident shock wave, reflected and refracted shocks are formed; the tangential discontinuity has a kink. In the other (Fig. 102c), a reflected rarefaction wave is formed, and a refracted shock wave transmitted into the other gas. Both can occur only in certain ranges of the parameters of the incident shock wave and the tangential discontinuity.[7]

The interaction of two tangential discontinuities may yield a configuration with no shock wave reaching it and two leaving it; in the absence of the tangential discontinuities, this is impossible, as shown above. In Fig. 103, the gas in region 1 is at rest; the configuration is, however, evidently possible only if there is supersonic flow in regions 2 and 5.

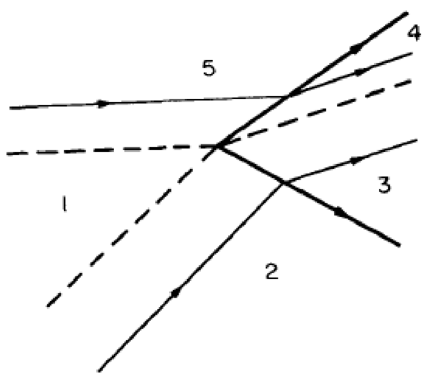

FIG. 103

We may briefly discuss the intersection of a shock wave with a weak discontinuity arriving from an external source. Here two cases can occur, according as the flow behind the shock wave is supersonic or subsonic. In the former case (Fig. 104a), the weak discontinuity is refracted at the shock wave into the space behind the latter; the shock itself is not refracted at the intersection, but has a singularity of a higher order, like that at a weak discontinuity.

[7] These two configurations are a kind of generalization of the cases shown in Figs. 100 and 101b.

Moreover, the entropy change in the shock wave must cause behind it a weak tangential discontinuity, at which the derivatives of the entropy are discontinuous.

If, however, the flow becomes subsonic behind the shock wave, the weak discontinuity cannot penetrate into this region, and it ceases at the point of intersection (Fig. 104b). The latter is now a singular point; for example, if the incident discontinuity relates to the first derivatives of hydrodynamic quantities, and the one leaving is a weak tangential discontinuity, it can be shown that the shock wave and the pressure distribution near the intersection have logarithmic singularities. Furthermore, as in the previous case, a weak tangential discontinuity of the entropy must occur behind the shock wave.[8]

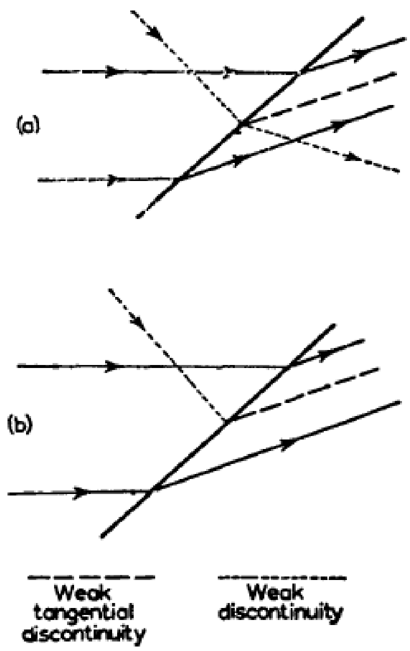

Fig. 104

The foregoing discussion of the interaction of shock waves with weak discontinuities applies also to that with weak tangential discontinuities. If the flow behind the shock wave is supersonic, a weak discontinuity and a weak tangential discontinuity are formed there; if it is subsonic, only a refracted weak tangential discontinuity occurs.

Finally, let us refer also to the interaction between weak and tangential discontinuities. If the flow on either side of the tangential discontinuity is supersonic, then reflected and refracted weak discontinuities are formed, in addition to the incident one; if there is subsonic flow beyond the tangential discontinuity, the weak discontinuity does not penetrate there, and undergoes "**total internal reflection**".

[8] A detailed quantitative analysis of the intersection of shock waves with weak discontinuities is given by S. P. D'yakov, Soviet Physics JETP 6, 729, 739, 1958.