§8. Spherical Detonation

Let us consider the disturbance in a gas due to a detonation at the centre of symmetry at time $t = 0$; a spherical detonation wave is propagated through the initially undisturbed gas when $t > 0$. We assume that heat changes occur only at the shock front; the gas motion is adiabatic beyond the shock, which is a detonation wave.

Let us first study the case when the density ρ_1 and the pressure p_1 are constant and non-zero in the initially undisturbed gas (Zel'dovich, 1942), The disturbed motion of a perfect gas is determined by the parameters

$$r, t, \gamma, \gamma_1, p_1, \rho_1, Q,$$

where Q is the heat liberated at the front by unit mass of gas; γ_1, γ are the appropriate values of the specific heat ratio; γ_1, ahead of the front and γ behind the front. The motion is **self-similar** and belongs to **type 1** defined in §5.

The family of integral curves in the z, V plane for the differential equation (5.3) ($\omega = 0$) is shown in Fig. 30; $\lambda = \dfrac{\beta r}{\sqrt{Q}t}$.

The gas is at rest in the region ahead of the detonation wave; consequently, the point H_1, on the z axis corresponds to the far side of the detonation wave in the z, V plane. At this point $\lambda_2 = \beta \dfrac{c}{\sqrt{Q}}$; we determine β from the condition $\lambda_2 = 1$. It follows from the Hugoniot condition (2.27) that the inner side of the detonation wave in the z, V plane corresponds to the parabola

$$z_2 = (1 - V_2)^2 \frac{1 + \gamma\Lambda}{1 - \Lambda}. \tag{8.1}$$

The Chapman- Jouguet condition is satisfied for $\Lambda = 0$ and the parabola (8.1) coincides with the parabola

$$z = (1 - V)^2. \tag{8.2}$$

If $\Lambda > 0$, the parabola (8.1) lies above the parabola (8.2) (see Fig.

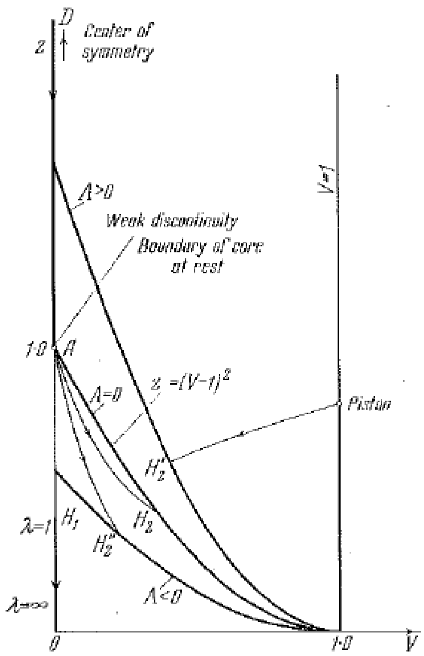

FIG. 42. Integral curves in the z, V plane corresponding to a spherical detonation. Case $p_1 = \text{const} \neq 0$, $\rho_1 = \text{const}$ ($\omega = 0$).

30 and the diagram on Fig. 42). Since the variable λ has a stationary value on the parabola (8.2}, it is impossible to extend the solution continuously between points of parabola (8.1) and the centre of symmetry. It is also easy to see that a solution with an additional compression shock is impossible.

However, the solution can be continued as far as the line $V = 1$; the points of this line can be considered to represent a spherical piston. Hence, the solution of the problem of piston expansion in a detonated mass of gas can be obtained. The choice of the appropriate integral curve and of the parameter Λ is fixed by the values of the pressure or velocity (λ^*) on the piston since the point H' on the parabola (8.1) (see Fig. 42) is determined for each Λ by the value of the parameter $\frac{p_1}{Q\rho_1}$.

If $\Lambda < 0$, then the parabola (8.1) is situated below the parabola (8.2). In this case, the solution of the problem is not unique.

There is a whole **pencil of curves** issuing from the node A which give solutions satisfying all the boundary conditions. The position of a point behind the shock front H''_2 and the parameter $\Lambda < 0$ are determined by finding the detonation front velocity. A core of gas at rest is obtained at the centre, whose boundary corresponds to the point A and which is a weak discontinuity. The line AD corresponds to points of the core. The point D corresponds to the core centre $\lambda = 0$ (Fig. 43). If the point H_2 lies on the parabola (8.2), then the Chapman-Jouguet condition is satisfied and the detonation velocity will be a minimum.

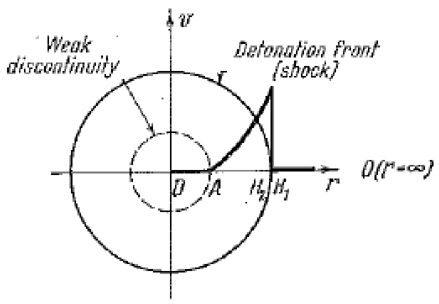

FIG. 43. Flow picture for a spherical detonation.

If $\Lambda \le 0$, then a **rarefaction wave** extends from the rear of the detonation front to the gas at rest.

Figures 44, 45 and 46 show the computed distributions of pressure, velocity and temperature in an example in which $\gamma = \gamma_1 = 5/3$, $p_1 = 0$ and the Chapman-Jouguet condition is satisfied (curves corresponding to $\omega = 0$).

The solution of the detonation problem in the cylindrical and plane wave cases can be obtained in a similar manner.

We now consider the problem of detonation in a medium with a variable initial density (Sedov, 1956; Iavorskaia, 1956).

If

$$0 < \omega \le \frac{2\gamma}{\gamma + 1},$$

then the field of integral curves is shown in Fig. 31.

Figure 47 shows the integral curves in the z, V plane relevant to this problem.

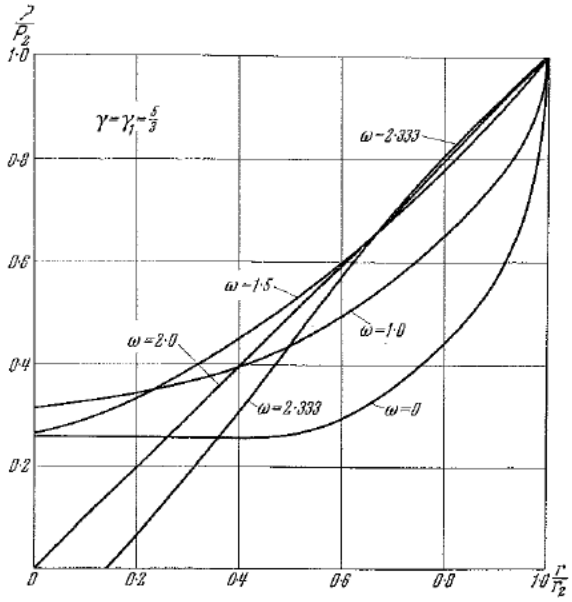

FIG. 44. Distribution of pressure behind the detonation wave front: initial pressure $p_1 = 0$, initial density $\rho_1 = A/r^{\omega}$; a vacuum is formed near the centre for $\omega > [3(\gamma+1)]/(3\gamma-1)$ within which the pressure is zero.

In order to continue the solution inwards behind the detonation wave front, it is necessary to use the integral curve starting from the point D at which $\lambda = 0$ and which passes through the singular point A. A weak discontinuity can arise at the point A on the parabola $z = (1-V)^2$; the variable λ has a finite value and increases as we move downwards along any integral curve; the only integral curves which yield a solution are those intersecting the parabola given by Equation (2.29), namely,

$$z_2 = \gamma V_2(1-V_2)\,. \tag{8-3}$$

Just as in the $\omega = 0$ case, the solution is not unique when $\Lambda \le 0$ on the detonation front. The solution satisfying the Chapman-Jouguet condition $\Lambda = 0$ corresponds to the points of intersection of the parabolas (8.2) and (8.3).

The solution for propagation of a detonation ware under the influence of a spherical piston expanding from the centre is given by

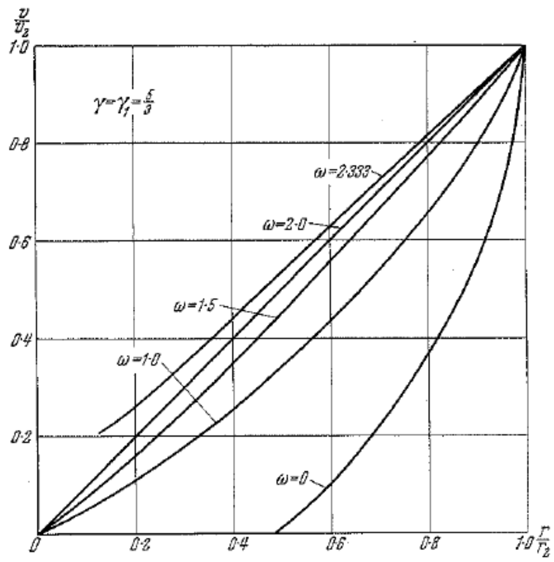

FIG. 45. Velocity distribution behind the detonation wave front ($p_1 = 0$, $\rho_1 = A/r^{\omega}$).

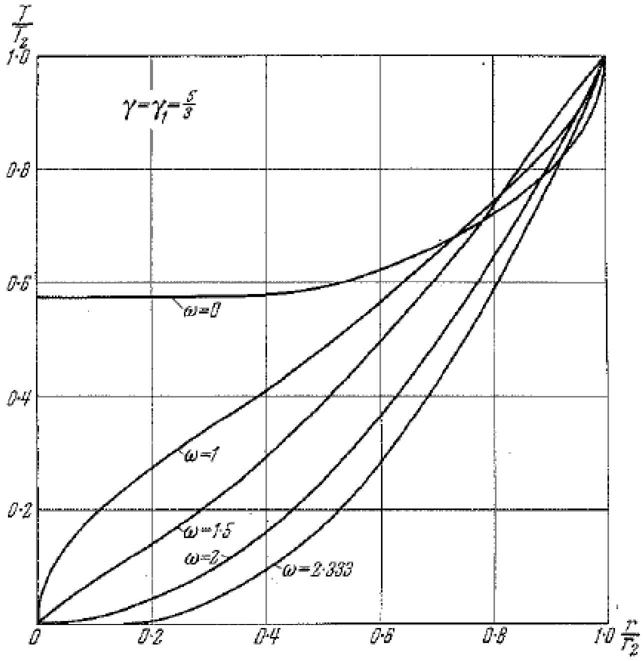

FIG. 46. Temperature distribution behind the detonation wave front ($p_1 = 0$, $\rho_1 = A/r^{\omega}$).

the integral curve of type $H''_2 C$, shown by the broken curve on Fig. 47.

Curves showing the variation of the gas motion characteristics are given on Figs. 44, 45 and 46 in the case when the Chapman-Jouguet condition is satisfied, $\gamma = 5/3$ and $\omega = 1.5$.

If $\omega \to \dfrac{2\gamma}{\gamma+1}$ then the singular point A is displaced along the parabola (8.2) and approaches the point of intersection with the parabola (8.3); the points H_2 and A coincide for $\omega = \dfrac{2\gamma}{\gamma+1}$; a unique solution is obtained in which the Chapman-Jouguet condition is satisfied.

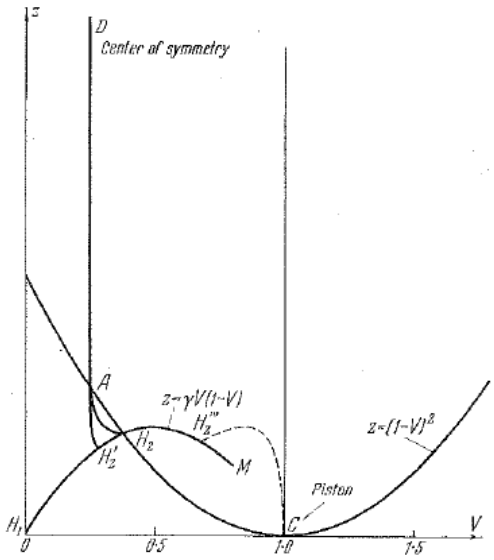

FIG. 47. The integral curves H_2AD or $H_2'AD$ correspond to the solutions of the detonation problem for $0 < \omega < [2\gamma/(\gamma+1)]$. The Chapman-Jouguet condition is satisfied at the point H_2.

If $\frac{2\gamma}{\gamma+1} < \omega < \frac{3(\gamma+1)}{3\gamma-1}$ there is no solution which will satisfy the Chapman-Jouguet condition.

The solution is unique and is furnished by the integral curve starting from the singular point D and intersecting the parabola (8.3) at the point H_2 for which $A > 0$ (Fig. 48).

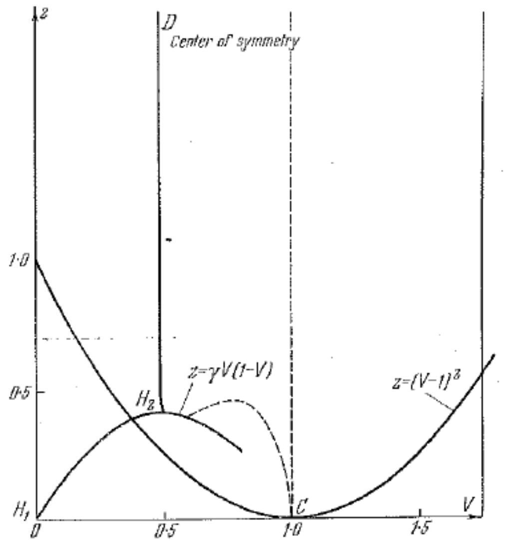

FIG. 48. The integral curve DH_2 corresponds to the solution of the detonation problem for $2\gamma/(\gamma+1) < \omega < [3(\gamma+1)]/(3\gamma-1)$. The Chapman-Jouguet condition is not satisfied.

If $\omega \to \frac{3(\gamma+1)}{3\gamma-1}$, then the singular point B (see Fig. 31) moves upwards, passes through the singular point A and then exchanges roles with this point. When $\omega = \frac{3(\gamma+1)}{3\gamma-1}$, the point B lies on the parabola (8.3) {see Fig. 32). In this case, it is necessary to make a jump from the point O to the point B in order to obtain the solution. All the gas motions behind the wave front in the z, V plane correspond to the single point B. In this case, $z = z_2 = \text{const}$, $V = V_2 = \text{const}$ behind the wave front, and the appropriate solution is given by the simple formulas

$$\frac{v}{v_2} = \frac{r}{r_2}, \quad \frac{\rho}{\rho_2} = \frac{r_2}{r}, \quad \frac{p}{p_2} = \frac{r}{r_2}. \tag{8.4}$$

The velocity behind the wave front is directly proportional to the coordinate r.

If $\frac{3(\gamma+1)}{3\gamma-1} < \omega$, then it is evident from Fig. 33 that the integral curve passing through the point A and subsequently through the point C corresponds to the solution in the z, V plane. An expanding sphere within which the pressure is zero corresponds to the singular point C. The value of A and, therefore, of the detonation velocity, is determined by the point of intersection of the integral curve with the parabola (8.3) (see Fig. 49).

The variation of the gas characteristics is given for $\gamma = 5/3$ and $\omega = 7/3 = 2.33$ in Figs. 44, 45 and 46.

Clearly all conclusions about increasing the detonation velocity

and on the formation of a vacuum at the centre of symmetry based on the self-similarity property, are independent of the magnitude of the heat liberated Q and only depend on the law under which the initial density falls off, and this is determined by the value of the exponent ω.

The increase in the detonation, velocity as compared with the velocity given by the Chapman-Jouguet rule can be obtained also for gas detonations in **tapered tubes**. If the cross-sectional area varies according to a power law, then we find, using the hydraulic approximation, that the gas motion is self-similar and is determined by (5.3), (5.4) and (5,5), but for $\nu < 1$. The magnitude of ν is determined by the law of stream tube contraction.

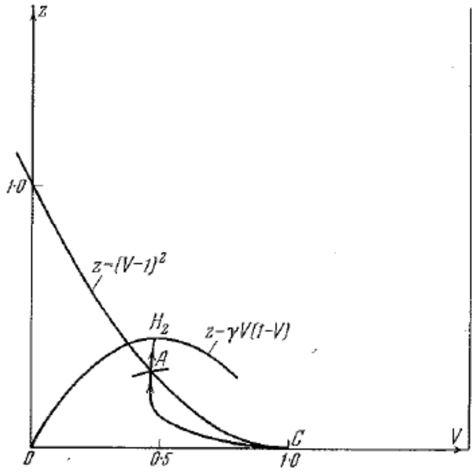

FIG. 49. The integral curve H_2AC corresponds to the solution of the detonation problem for $\omega > [3(\gamma+1)]/(3\gamma-1)$. The point C corresponds to an expanding vacuum. The Chapman-Jouguet condition is not satisfied.