§9. Flame Propagation

We consider the disturbed motion of a combustible mixture resulting from combustion in a very thin moving layer. Analysis[1] shows that the thickness of the layer in which the chemical reaction occurs can be neglected in a number of cases and we arrive at the problem of gas motion in which the chemical reaction and the heat liberation are performed instantaneously at a certain surface, across which variables characterizing the state and motion of the gas vary discontinuously; the surface is called a ***flame front***. In contrast to the **detonation front**, the flame front is a **rarefaction jump**, the velocity of propagation the flame front, u, through the combustible mixture is a known chemical constant. The flame propagation velocity is small in comparison with the speed of sound and, therefore, in comparison with the detonation velocity.

Conditions (2.12), (2.13) and (2.14) are satisfied on the flame front, just as on the detonation front; the difference between the flame and detonation fronts is only that the *flame front velocity is small and known in advance*. Disturbances caused by combustion are propagated into the gas in front of and behind the flame front. It is necessary to take the smaller root $\rho_1/\rho_2 > 1$ as the solution of (2.14). This corresponds to states which are obtained from the initial state with continuous heat liberation (without a heat absorption zone). The reaction thus occurs in a thin, finite layer.

We consider the problem of propagation of a **plane** flame front through a gas at rest with density ρ_1 and pressure p_1 in a cylindrical tube. Burning progresses towards a closed end of the tube. This solution of the problem is very simple and is as follows: a shock wave moves from the closed end through the undisturbed gas; behind the shock wave front the gas motion is directed forwards toward the shock wave. A plane flame front is propagated through the moving gas leaving the gas behind it at rest, a consequence of the boundary conditions at the closed end. In order to solve the problem completely, it is sufficient to write and solve **six equations** simultaneously; three on the flame front and three at the shock. The six unknowns to be determined from the six equations are: the density and pressure behind the flame front and behind the shock, the gas velocity behind the shock wave and the velocity of shock wave propagation.

Consider the problem of a **spherical** flame front under the assumption that combustion starts at $t = 0$ at a point, is then propagated by means of a spherical wave through the undisturbed gas with constant density ρ_1 and constant pressure p_1. Evidently, the disturbed gas motion is self-similar and is determined by the same constants as in the detonation phenomenon. The integral curves in the z, V plane for a spherical flame are given in Fig. 30 just as for the spherical detonation case.

The gas particles sufficiently far removed from the ignition centre at any instant $t > 0$ will be at rest. The stationary region corresponds to the integral line $V = 0$. It is impossible to effect the transition from rest, $V = 0$, to motion on another integral curve in the left part of the $z > 0$, $V < 0$ half-plane by means of a rarefaction jump, a flame front through the singular point A with a weak discontinuity, since the subsequent motion cannot be continued to the centre of symmetry. In these cases, continuous motion or motion in the presence of a shock follows an

[1] The problem of spherical flame propagation when $\rho_1 = \text{const}$ and $p_1 = \text{const}$ was studied by G. M. Bam-Zelikovich.

integral curve intersecting the parabola $z=(1-V)^2$.

Therefore, the transition from rest, V = 0, to motion on another integral curve is only possible by means of a transition through a simple compression shock originating at $z_1<1$. According to (2.10), compression shocks transform the V = 0 axis into points of the parabola.

$$z_2=(1-V)\left(1+\frac{\gamma-1}{2}V_2\right). \qquad (9.1)$$

To continue the solution up to the centre of symmetry where V = 0, the flame front must be so determined that a point behind it would be located either on the integral line V = 0 leaving the singular point D ($z=\infty$, V = 0) or on the integral curve L entering the singular point B ($z=\frac{3(\gamma-1)^2}{(3\gamma-1)^2}$, $V=\frac{2}{3\gamma-1}$)[2] (see Fig. 50). It follows from condition (2.25) that transition through the flame front is possible on the V = 0 axis only from points of the curve

$$z_3=\frac{V_3(1-V_3)\left(1+\frac{\gamma'-1}{2}V_3\right)+(\gamma'-1)\frac{Q}{u^2}(1-V_3)^3}{\frac{\gamma'}{\gamma}-\frac{\gamma'-1}{\gamma-1}(1-V_3)}. \qquad (9.2)$$

Equation (9.2) has been obtained from (2.25) after replacing the subscripts 1, 2 by 4, 3, putting $V_4=0$ and changing the sign in $\frac{Q}{c^2}=\frac{Q(1-V_3)^2}{u^2}$ since combustion occurs as the transition is made from state 3 to state 4; u is the flame propagation velocity:

$$u=c-v=\frac{r}{t}(1-V_3)=c(1-V_3).$$

In (9.2), γ refers to the moving combustible mixture; γ' to the combustion products at rest.

Figure 50 shows the method of constructing the solution.

The image point appears to be at a certain point $zl2$, V_2 on the parabola (9.1) following the jump; movement along the integral curve passing through this point in the direction of decreasing λ corresponds to further motion toward the centre, i.e., motion in a region above the parabola (9.1). The point of intersection of the integral curve under consideration with the curve (9,2) corresponds to the leading edge of the flame front. Behind this there is a stationary core of gas, corresponding to points of the V = 0 axis.

The construction described is always possible since any integral curve starting from the parabola. (9.1) intersects the curve. (9.2) for $V\le\frac{2}{\gamma+1}$ and a transition is possible from any point of the curve (9.2) to points of the V = 0 axis through the flame front.

However, we assume in this construction that $z_4\ge1$. Points of the curve (9.2) for which $z_4<1$ is obtained cannot correspond to the

[2] If the initial density varies according to the law $\rho_1=A/r^{\omega}$, then the coordinates of the point B depend on ω, see §5.

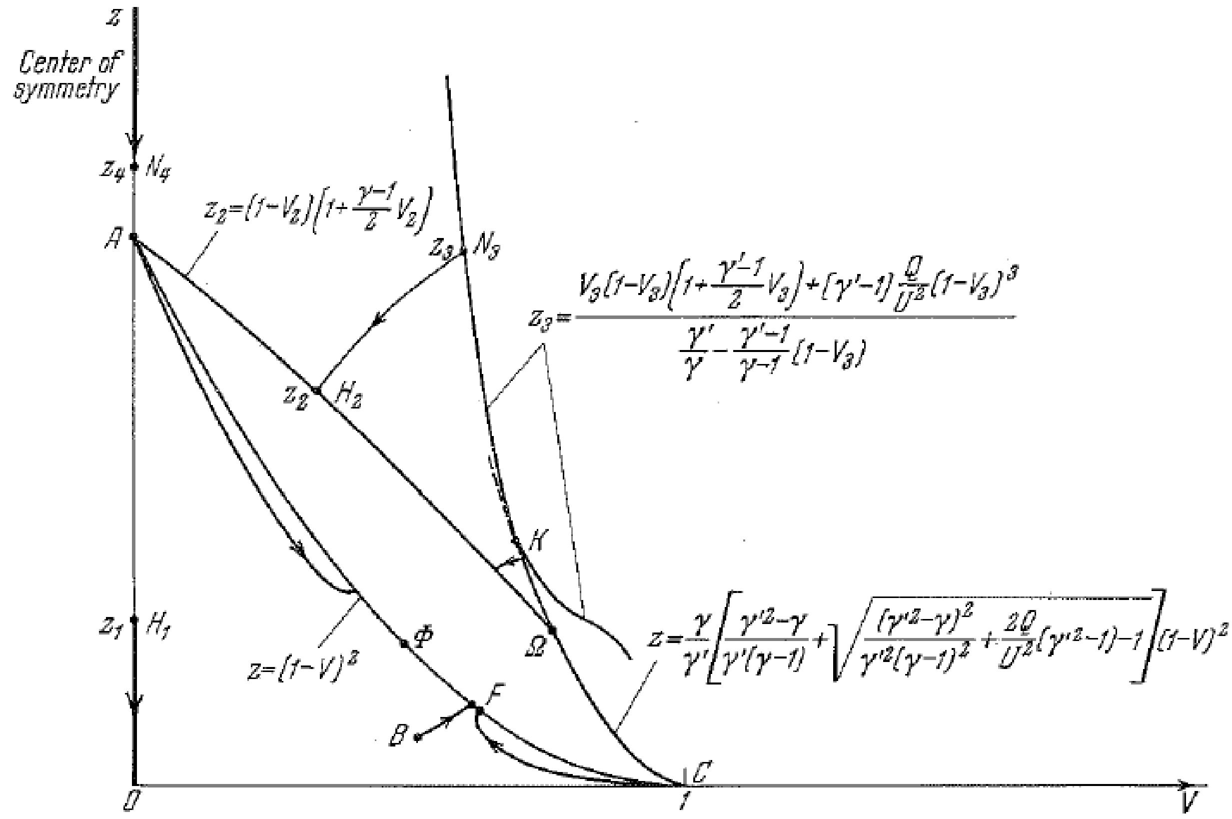

FIG. 50. Integral curves in the z, V plane corresponding to spherical combustion. The points H_1 and H_2 correspond to the shock wave ahead of the flame front. The points N_3 and N_4 correspond to the flame front.

leading edge of the flame front since this leads to a supersonic flame front velocity relative to the particles behind the front. In this case, the flame front can be constructed by using a jump on to the parabola $z=(1-V)^2$ from the point of intersection of the integral curve for the gas motion behind the shook wave, with the parabola[3]

$$z=\frac{\gamma}{\gamma'}\left\{\frac{\gamma'^2-\gamma}{\gamma'(\gamma-1)}+\sqrt{\frac{(\gamma'^2-\gamma)^2}{\gamma'^2(\gamma-1)^2}+\frac{2Q}{u^2}(\gamma'^2-1)-1}\right\}(1-V)^2 \tag{9.3}$$

(Fig. 50, curve $K\Omega C$). The propagation velocity of such a jump through the gas behind the front exactly equals the speed of sound, while an additional rarefaction wave is formed behind the front. This rarefaction wave corresponds to an integral curve proceeding from a point on the parabola $z=(1-V)^2$ to either the point A, which corresponds to the boundary of the core at rest, or from the point F to the point B, in which case a core at rest is not formed, and the motion can be continued up to the centre of symmetry, or to the singular point C ($z=0$, $V=1$); a vacuum is formed near the centre in the latter case.

A solution of the problem exists for all points z_2, V_2 on the parabola (9.1) located above the point Ω at which the parabola (9.1) intersects the parabola (9.3). The point Φ corresponds to the point

[3] The parabola (9.3) transforms into the parabola $z=(1-V)^2$ for a, jump of the flame front.

Ω on the parabola $z=(1-V)^2$. If the point Φ lies above the point F, then a stationary core is always formed near the centre of symmetry. If p_1 and ρ_1 are constant then a stationary gas core occurs near the centre of symmetry. If $\rho_1 = A/r^{\omega}$, a vacuum can form near the centre of symmetry for a certain value of ω.

The solution of problems of propagation of cylindrical flames can be constructed by using analogous methods. The results of numerical computations and a comparison of various cases are given in Figs. 51, 52 and 53.

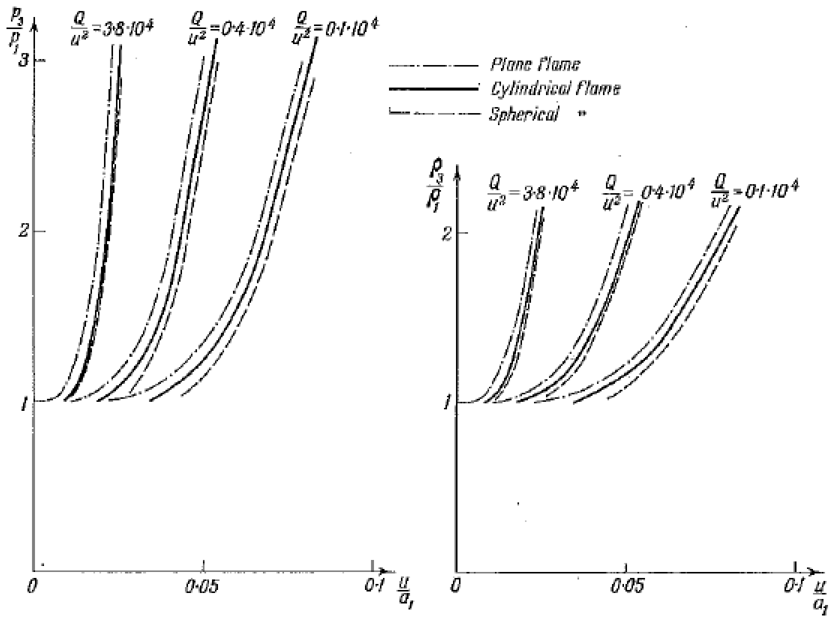

FIG. 51. Density and pressure ahead of the flame front for various amounts of heat released Q/u^2 (Q is the energy liberated per unit mass; u is the flame front velocity through the gas; p_1 is the pressure; ρ_1 is the density and a_1 is the speed of sound in the combustible mixture.

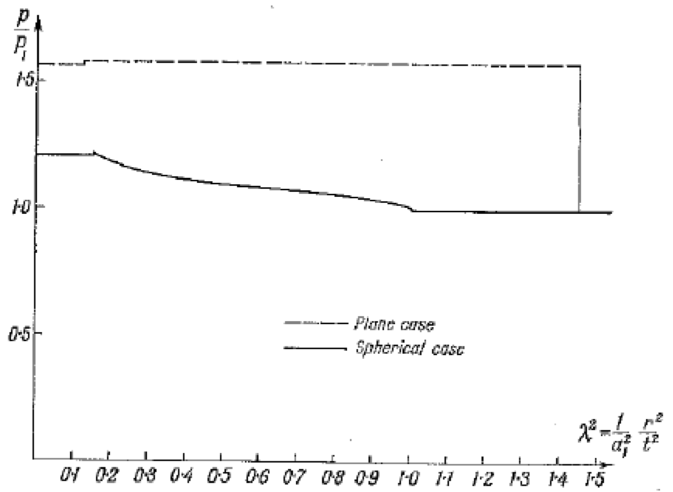

FIG. 52. Pressure distribution for propagation of combustion from a plane wall (plane flame front) and from a point (spherical flame front): $Q/a_1^2 = 60$ and $u/a_1 = 0{\cdot}016$.

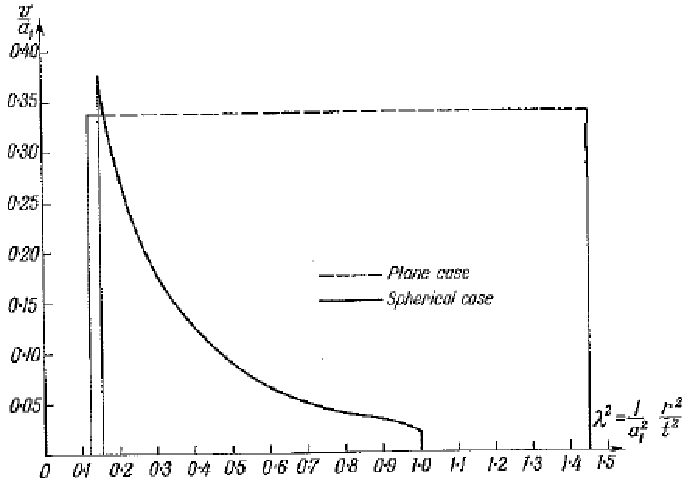

FIG. 53. Velocity distribution for propagation of combustion from a plane wall (plane flame front) or from a point (spherical flame front): $Q/a_1^2 = 60$; and $u/a_1 = 0 \cdot 016$.