§10. Collapse of an Arbitrary Discontinuity in a Combustible Mixture

We now discuss, in outline only, the general character of the problem of the collapse of an arbitrary discontinuity (formulated in §1). Detailed analysis will be omitted.[1]

We first consider two inert gases separated by a surface of discontinuity and assume that the pressure in the second gas to the left is larger than that in the first gas to the right of this surface (the converse case is exactly similar). Then, if the x axis runs from left to right and if the difference $v_1 - v_2$ of the initial gas velocities is negative and large in absolute value (this case will occur, for example, if both initial gas velocities are directed toward the surface of discontinuity), then shock waves will develop on both sides of the discontinuity. At the gas interface there is a stationary discontinuity at which the pressure and the normal velocity are continuous but the density changes discontinuously. A graph of the pressure in this case is shown in Fig. 54.

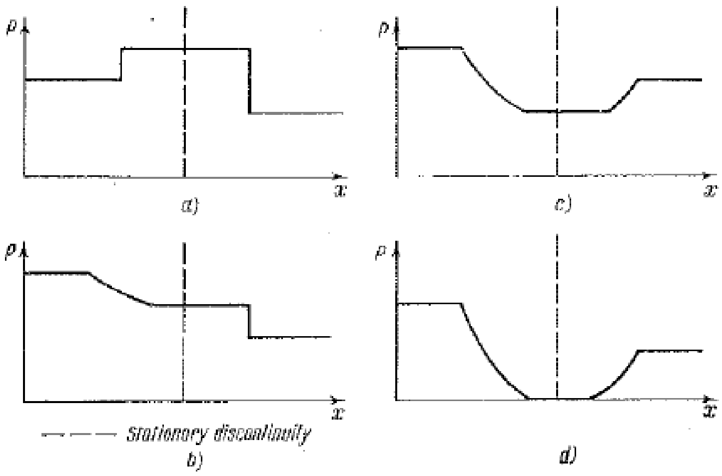

Fig. 54. Various forms of decay of an arbitrary discontinuity in an inert gas.

As the difference between the initial velocities increases, the shock wave in the second gas changes to a rarefaction wave (Fig. 54b) and then a rarefaction wave in the first replaces the shock wave (Fig. 54c). When the initial velocity difference becomes a very large positive quantity, a vacuum forms between the rarefaction waves on both sides (Fig. 54d).

A more complex case arises when a combustible mixture is on the right of the surface of discontinuity so that a flame front can develop when the surface collapses. The general features of the motion which occurs in this case will be analogous to that considered above.[2]

When the initial velocities differ by a small amount a shock wave is propagated through the inert gas, a shock wave also develops in the combustible mixture followed by a flame front. A stationary discontinuity can exist between the inert gas and the combustion products.

A graph of the pressure in this case is shown in Fig. 55a (the flame front is denoted by the wide vertical band, the stationary discontinuity by dashes). As the difference in the initial velocities increases, the

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[1] These problems are analysed in detail in Landau and Lifshitz (1959).

[2] This question was worked out quantitatively in detail by Bam-Zelikovioh (1949).

shock wave in the inert gas first changes into a rarefaction wave (Fig. 55b) and then a rarefaction wave arises ahead of the flame front in place of the shock wave (Fig. 55c), Here, the velocity of the combustion products relative to the flame front increases just until sonic speed is attained.

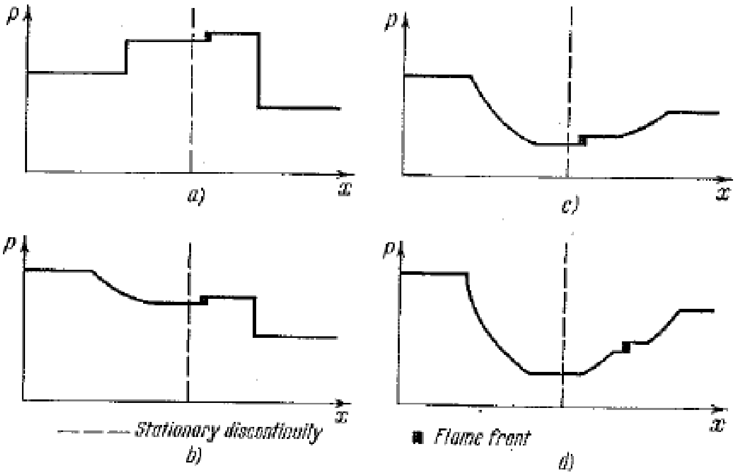

Fig. 55. Various cases of collapse of a surface of discontinuity in a combustible mixture.

As the difference in the initial velocities increases further, the flow ahead of the flame front does not change but still another rarefaction wave appears directly behind the front (Fig. 55d). A vacuum can form between the inert gas and the combustion products if the difference in the initial velocities is very large.

If the pressure in the inert gas is less than in the combustible mixture, a rarefaction wave may develop in the combustible mixture ahead of the flame front but there is a shock wave in the inert gas.

Similarly, if there is a **detonation wave** in the combustible mixture, then by increasing the difference in the initial velocities from $-\infty$ to $+\infty$, we find that there is first a shock wave in the inert gas and a detonation wave with velocity as large as desired in the combustible mixture; then the shock wave in the inert gas is replaced by a rarefaction wave and the velocity of the detonation wave decreases to a certain definite value. Here the velocity of the detonation products relative to the front increases until sonic speed is attained. Later, the detonation wave velocity varies and a rarefaction wave forms behind it.

We now record the various types of formation and collapse of an arbitrary discontinuity.

(1) A shook wave is propagated into the gas and a **second shock wave overtakes it** from behind. At the instant of overtaking, a surface of discontinuity is formed across which the conditions of conservation of mass, momentum and energy are not satisfied, i.e., an arbitrary discontinuity is formed.

Computations show that shock waves will develop on both sides in this case after the discontinuity has collapsed.

(2) A shock wave approaches the interface of two media of different densities. When the shock wave crosses from one medium into the other an arbitrary discontinuity is formed. Two types of motion are possible when this discontinuity collapses.

Shock waves will occur on both sides when the wave crosses from

the less dense to the more dense medium (for example, from air to water). If the wave crosses from the more dense to the less dense medium (from water to air, say) then a shock wave occurs in the front (in the air) and a rarefaction wave in the rear (in the water).
(3) A low intensity shock wave overtakes a flame front. (This case is encountered in **pulsating combustion** in closed vessels.) After the wave has overtaken the flame, shock waves will occur on both sides of the flame front. If a low intensity shock wave encounters a flame front, then a rarefaction wave will occur ahead of the flame front after the arbitrary discontinuity has collapsed and a shock wave will develop in the combustion products.

The problem of the collapse of a given discontinuity is important in the study of the initial stages of gas motion in shock tubes. A diagram of the motion in a shock tube is pictured on Fig. 56. Two gases of high and low pressure are separated by a membrane. An arbitrary discontinuity forms after the membrane is suddenly destroyed; consequently, a shock wave occurs in the low pressure gas. The high pressure gas is either at rest or is moving at the moment the membrane bursts if a shook or detonation wave approaches the membrane. The shook wave intensity in the low pressure gas depends on the initial motion and on the pressure drop, on the difference in the temperatures and on the properties of the gases initially separated by the membrane.

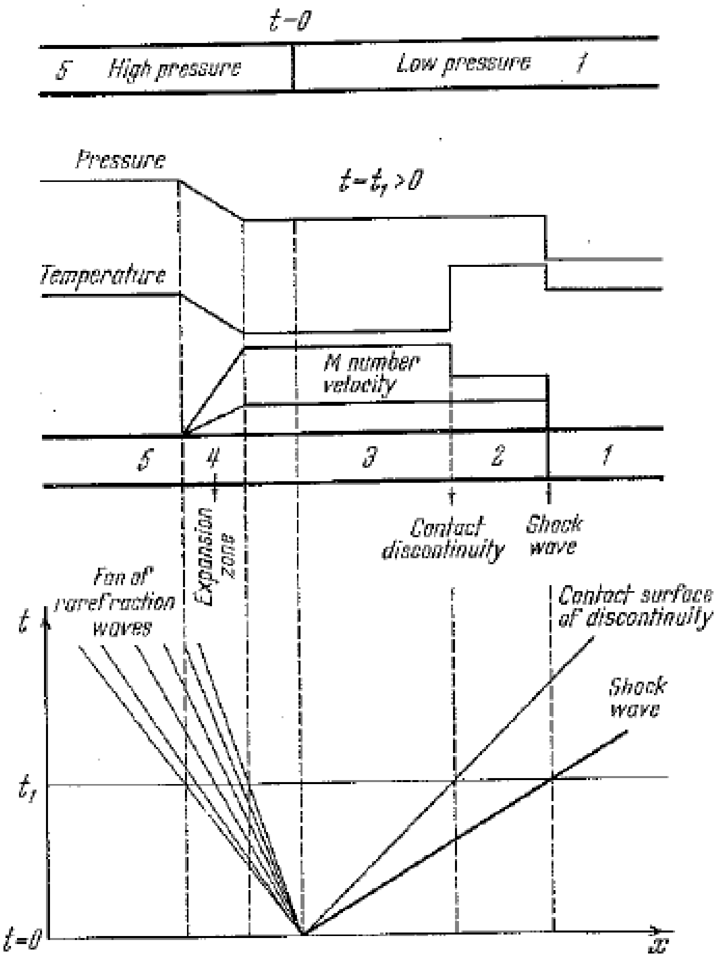

FIG. 56. Diagram of the motion in a shock tube.

The shock wave intensity will increase, other conditions remaining equal, if a gas with a reduced initial sonic speed is used as the low pressure gas. In polyatomic gases the reduction in sound speed can be achieved by using a gas with reduced γ.

For example, $\gamma = 1.67$ and the sound speed is $a_1 = 975$ m/sec for helium at a 273 *K* temperature; $\gamma - 1.4$ and $a_1 = 333$ m/sec for air and $\gamma = 1.15$ and $a_1 = 121.5$ m/sec for freon.

Other conditions being equal, the shock wave intensity in the low pressure gas increases as its temperature decreases.

The shock wave intensity in the low pressure gas is evidently very much larger than the shock wave intensity approaching the membrane in the high pressure gas.

Very intense shock waves, with high temperatures behind the wave front and high speed gas motions can be obtained in shock tubes.

Particles with a high temperature, which drops rapidly within the short time τ, are obtained behind the shock wave front.

Shock tubes are used widely for aerodynamic investigations of very high speed flows around bodies. They are also used in physical chemistry investigations, in particular, to obtain chemical reactions at high temperatures. The opportunity to achieve high temperatures during very short time intervals permits the kinetics of chemical reactions to be studied and intermediate products in chain reactions to be obtained.