§11. Problem of an Intense Explosion[1]

1. Intense explosion in a gas

The above arguments allow that, in an intense explosion the disturbed air region is separated from the undisturbed air by a shock wave.

As already mentioned, the pressure ahead of the shook wave can be neglected in comparison with the pressure behind the shock wave in an intense explosion. Let us first estimate with what accuracy and for which shock waves this statement is valid.

Using the property $v_1 = 0$, we rewrite the shock conditions (2.5) and)2.6) as follows:

$$\left.\begin{aligned} v_2 &= \frac{2}{\gamma+1} c\left[1-\frac{a_1^2}{c^2}\right] = \frac{2c}{\gamma+1} f_1 \\ \rho_2 &= \frac{\gamma+1}{\gamma-1}\rho_1\left[1+\frac{2}{\gamma-1}\frac{a_1^2}{c^2}\right]^{-1} = \frac{\gamma+1}{\gamma-1}\rho_1 f_2 \\ p_2 &= \frac{2}{\gamma+1}\rho_1 c^2\left[1-\frac{\gamma-1}{2\gamma}\frac{a_1^2}{c^2}\right] = \frac{2}{\gamma+1}\rho_1 c^2 f_3 \end{aligned}\right\} \qquad (11.1)$$

where c is the velocity of shock wave propagation.

As the shock wave increases in strength the ratio a_1 / c is reduced.

Figure 57 shows f_1, f_2 and f_3 as a function of a_1 / c. Figure 58 shows p_2 / p_1 as a function of the ratio a_1 / c for $\gamma = 1.4$. We observe that the quantities f_1, f_2 and f_3 differ from unity by less than 5 per cent when $a / c_1 < 0.1$.

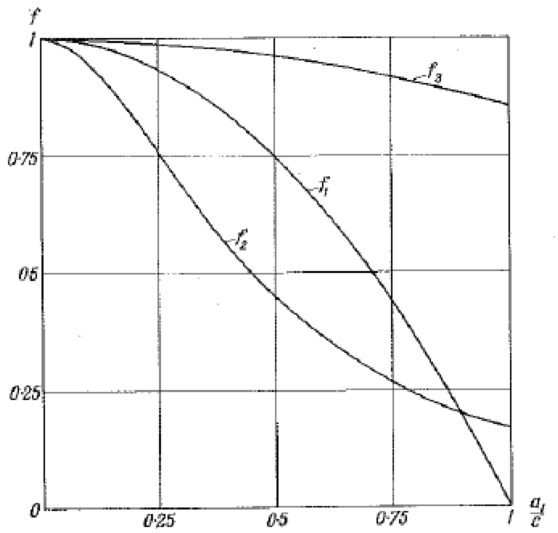

FIG. 57. Relation between the quantities f_1, f_2 and f_3 and the ratio a_1/c, where a_1 is the speed of sound in the undisturbed medium; c is the shock wave velocity.

If we put $a / c_1 = 0$ and $f_1 = f_2 = f_3 = 1$ into (11.1) (or if we put $p_1 = 0$, which is equivalent), then an error of less than 5 per cent is introduced hi the values of v_2, ρ_2 and p_2.

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[1] In this section, we shall explain the exact theoretical formulation and the numerical solution of the problem of an intense explosion for both spherical and cylindrical and plane waves, This was first published in Sedov (1946a) and in Sedov (1946b).

The conditions on the shock wave then become

$$\left.\begin{aligned} v_2 &= \frac{2}{\gamma+1}c \\ \rho_2 &= \frac{\gamma+1}{\gamma-1}\rho_1 \\ p_2 &= \frac{2}{\gamma+1}\rho_1 c^2 \end{aligned}\right\}. \qquad (11.2)$$

The velocity of shock wave propagation c is a characteristic parameter.

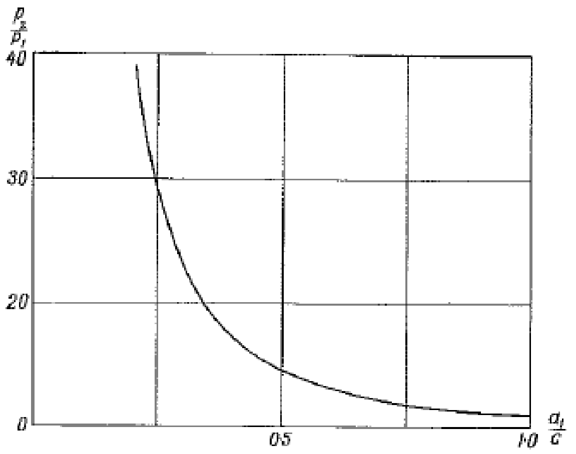

FIG. 58. Pressure drop across the shock wave as a function of the ratio a_1/c.

If we use the equations of motion in the form (1.3) and this formulation of the strong explosion problem, then we can take as fundamental dimensional constants:

$$\rho_1 \text{ and } E/\rho_1$$

where E is a certain constant which we shall determine later, and has the same dimensions as the energy E_0 liberated during the explosion; the dimensions of E are

$[E] = ML^2T^{-2}$ in the **spherical** case,

$[E] = MLT^{-2}$ in the **cylindrical** case,

$[E] = MT^{-2}$ in the **plane** case,

All three cases can be combined in the one formula

$$[E] = ML^{\nu-1}T^{-2},$$

Evidently, the constant E is directly proportional to E_0:

$$E_0 = \alpha E$$

where α is a constant.

In this case, the single nondimensional variable parameter λ is given by

$$\lambda = \frac{r}{\left(\dfrac{E}{\rho_1}\right)^{1/(2+\nu)} t^{2/(2+\nu)}}.$$

The motion of the shock wave is easily determined without solving the equations of motion.

Different equations of motion can be used provided that these do not contain new essential physical constants with dimensions independent of ρ_1

and E. In particular, it is not necessary to assume that the coefficient $\gamma = c_p / c_v$ in (1.3) is constant.

The shock wave coordinate r_2 is a function of the time t and since it is impossible to form a nondimensional combination of the dimensional quantities t, ρ_1 and E, then

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/(2+\nu)} t^{2/(2+\nu)} \lambda^*, \tag{11.3}$$

where $\lambda^* = \text{const}$; λ^* can be set equal to any non-zero number and the value of E can be calculated, from the magnitude of the charge energy E_0. Later, to be definite, and for the sake of simplicity, we shall set $\lambda^* = 1$. The constant α in the formula $E_0 = \alpha E$ is then determined from the solution of the equations of motion.

Hence, in the **spherical symmetry** case, the motion of the shock wave is given by

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/5} t^{2/5}, \quad c = \frac{2}{5}\left(\frac{E}{\rho_1}\right)^{1/5} t^{-3/5} = \frac{2}{5}\sqrt{\frac{E}{\rho_1}}\frac{1}{\sqrt{r_2^3}}; \tag{11.4}$$

and in the **cylindrical symmetry** case, by

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/4} \sqrt{t}, \quad c = \frac{1}{2}\left(\frac{E}{\rho_1}\right)^{1/4} \frac{1}{\sqrt{t}} = \frac{1}{2}\sqrt{\frac{E}{\rho_1}}\frac{1}{r_2}; \tag{11.5}$$

while for **plane** waves

$$r_2 = \left(\frac{E}{\rho_1}\right)^{1/3} t^{2/3}, \quad c = \frac{2}{3}\left(\frac{E}{\rho_1}\right)^{1/3} t^{-1/3} = \frac{2}{3}\sqrt{\frac{E}{\rho_1}}\frac{1}{\sqrt{r_2}}; \tag{11.6}$$

These formulas show that the law of shock wave attenuation depends on the charge shape.

The formula obtained above in the spherical symmetry case is in good agreement with published experimental results in photographs of an atomic bomb explosion in New Mexico in 1945.

Photographs of an atomic bomb explosion published in a paper by G. I. Taylor (1950) are given in Figs. 59, 60 and 61.

The air temperature is very high in the disturbed air region at quite significant distances from, the centre of the atom bomb explosion. Consequently, this region is shown as a luminous spot on the photographs. The boundary in the upper part of the spot is spherical, sharply traced and coincides with the shock wave. The shock wave attenuates as time increases and the temperature behind its front decreases. However, the appearance of the wave front is retained in its initial form because of the jump in the density. A relation between the radius r_2 of the expanding spherical shock wave and corresponding time t measured from the instant of initiation is derived from these photographs. The radius ranges from values of 11 to 185 m. at corresponding time intervals between 0.1×10^{-3} and 62×10^{-3} sec.

The experimental results are shown on Fig. 62 by crosses. The line corresponds to the formula

$$\frac{5}{2}\log r_2(cm) - \log t(\text{sec}) = 11.915$$

which agrees with the theoretical formula (11.4) after taking the logarithm of the latter, if it is assumed in addition that

$$\frac{1}{2}\log\frac{R}{\rho_1} = 11.915$$

from which for $\rho_1 = 0.00125$ (gm/cm^3)

$$E = 6.76 \times 10^{23} \rho_1 = 8.45 \times 10^{20} \text{ (erg)} \qquad (11.7)$$

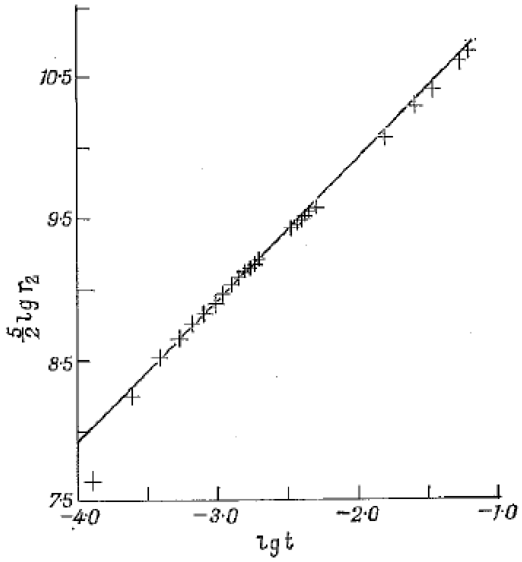

FIG. 62. Experimental results, shown by crosses, lie on a line inclined at 45° to the coordinate axes which is good confirmation of the theoretical formula $r_2 = (E/\rho_1)^{1/5} t^{2/5}$.

The experimental results are in good agreement with the law of shock wave propagation (11.4) established earlier by using the general reasoning of dimensional analysis and they permit the magnitude of the constant E to be determined according to (11.7).

The formula for the shock wave velocity can be written

$$c = \frac{2}{\nu + 2} \frac{r}{t}.$$

Substituting this expression for c into the shock condition (1.2) and transforming to nondimensional variables V, R, P and $z = \gamma P / R$ according to (1.1), we find the values V_2, R_2 and z_2 behind the shock wave

$$V_2 = \frac{4}{(\gamma+1)(\nu+2)}, \quad R_2 = \frac{\gamma+1}{\gamma-1}, \quad z_2 = \frac{8\gamma(\gamma-1)}{(\gamma+1)^2(\nu+2)^2}. \qquad (11.8)$$

The analysis of the family of integral curves in the z, V plane, given in §5, and the finite integrals established in §3, can be used to determine the field of disturbance due to an intense explosion.

The integral curves are similar for $\nu = 1,2$ to the fields of the integral curves in the spherical case for $\nu = 3$. This field is mapped in Fig. 34.

The parameter λ can only approach infinity as the singular points O and F are approached during continuous motion along the integral curve. Therefore, for continuous motion only the points O and F in the z, V plane can correspond to the points at infinity in the gas. The value $\lambda = 0$ corresponds to the centre of symmetry for $t \neq 0$. The parameter λ approaches 0 only along the integral curves $z = 0$ and $V = \pm\infty$ and when the singular point C is approached along the particular integral curve which originates at B. Finite values of the parameter $\lambda \neq 0$ correspond to the singular points B and D.

It is not difficult to see that a single integral curve terminating at the

singular point C corresponding to the centre of symmetry gives the solution of the problem of an intense explosion.[2] However, this solution cannot possibly be extended continuously to $r = \infty$, so that continuous gas motion is impossible in a violent explosion. In order that this solution can be continued and joined with the undisturbed solution through the strong shock wave, it is necessary and sufficient that the point M with the coordinates

$$V = \frac{4}{(\gamma+1)(\nu+2)}\,, \quad z_2 = \frac{8\gamma(\gamma-1)}{(\gamma+1)^2(\nu+2)^2}\,,$$

be an integral curve terminating at infinity at the singular point C, according to {11.8).

The important theoretical question of the existence of a solution of the violent explosion problem is related to the proof that two points M and C belong to the same integral curve of the first order ordinary differential equation (5.10) at $\omega = 0$, $\delta = 2/(2+\nu)$. It is evident that this question cannot possibly be solved by using the approximate numerical solution.

However, it turns out that the solution can be obtained in finite, closed form. In fact, it follows from the shock condition (11.8) that the constant on the right side of the energy integral (3,11) equals zero. Hence, the integral curve in the z, V plane corresponding to the desired solution is represented by the simple equation

$$z = \frac{(\gamma-1)V^2\left(V - \dfrac{2}{\nu+2}\right)}{2\left[\dfrac{2}{(\nu+2)\gamma} - V\right]}\,. \qquad (11.9)$$

Substituting z from (11.9) into (5.11) and using the boundary condition $\lambda = 1$ for $V = V_2 = \dfrac{4}{(\gamma+1)(\nu+2)}$, we find $\lambda(V)$ by means of a simple quadrature.

The function $R(V)$ is determined easily from the adiabatic integral and the functions $z(V)$ and $\lambda(V)$ can then be found. The constant in the adiabatic integral is determined by the shock conditions (11.8).

The integral curve (11.9) is the unique integral curve of (5.10) which terminates at the singular point C($z = \infty$, $V = \dfrac{2}{\gamma(\nu+2)}$). The variable λ decreases along this curve from the value unity at the shock wave to zero at the singular point C, where V has the finite value $\dfrac{2}{\gamma(\nu+2)}$. Hence, it follows that the gas velocity equals zero at the centre of symmetry ($v = \dfrac{r}{t}V$), which is a natural mechanical condition for the continuation of the solution to the centre of symmetry.

It has thus been proved that the solution of the self-similar problem of a violent explosion exists and is unique.

This proof is related essentially to the fact that the values of z_2 and V_2 (11.8) behind the shock wave front, belong to the same single integral curve which passes through the singular point C; consequently, the solution can be continued to the centre of symmetry on all the adjacent integral curves.

Standard formulas, tables and graphs are given below which are valid for $p_1 = 0$ for any values of the initial density ρ_1 and explosive energy E_0.

We shall use the following definitions and relations which result from the definition (1.1) and from the conditions on the shock wave (11.2):

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[2] A detailed analysis of this question is given in Sedov(1945a,b). See also Sedov (1946a).

$$\left.\begin{aligned}
&\frac{r}{r_2}=\lambda\\
&\frac{v}{v_2}=f=\frac{(\nu+2)(\gamma+1)}{4}\lambda V\\
&\frac{\rho}{\rho_2}=g=\frac{\gamma-1}{\gamma+1}R\\
&\frac{p}{p_2}=h=\frac{(\nu+2)^2(\gamma+1)}{8\gamma}\lambda^2 RZ
\end{aligned}\right\} \qquad (11.10)$$

in which.

$$\left.\begin{aligned}
&v_2=\frac{4}{(\nu+2)(\gamma+1)}\left(\frac{E}{\rho_1}\right)^{1/(2+\nu)}\frac{1}{t^{\nu/(\nu+2)}}=\frac{4}{(\nu+2)(\gamma+1)}\left(\frac{E}{\rho_1}\right)^{1/2}\frac{1}{{r_2}^{\nu/2}}\\
&\rho_2=\frac{\gamma+1}{\gamma-1}\rho_1\\
&p_2=\frac{8\rho_1}{(\nu+2)^2(\gamma+1)}\left(\frac{E}{\rho_1}\right)^{2/(2+\nu)}\frac{1}{t^{2\nu/(\nu+2)}}=\frac{8E}{(\nu+2)^2(\gamma+1)}\frac{1}{{r_2}^{\nu}}\\
&T_2=\frac{p_2}{\boldsymbol{R}\rho_2}
\end{aligned}\right\}$$

(11.11)

Let us denote initial coordinates of the gas particles in a Lagrangian system by r_0. It is evident that the coordinate r_0 for each particle equals the radius of the shock wave r_2 at the moment it passes through the particle.

Prom the shock conditions and from the adiabatic condition behind the wave front, it follows that

$$\frac{p}{\rho^{\gamma}}=\frac{p_2}{{\rho_2}^{\gamma}}(r_0)=\frac{8(\gamma-1)^{\gamma}}{(\nu+2)^2(\gamma+1)^{\gamma+1}}\frac{E}{{\rho_1}^{\gamma}}\frac{1}{{r_0}^{\nu}}. \qquad (11.12)$$

Hence, we obtain the formula

$$\left(\frac{r_0}{r_2}\right)^{\nu}=\frac{8\gamma(\gamma-1)^{\gamma}}{(\nu+2)(\gamma+1)^{\gamma+1}}\frac{R^{\gamma-1}}{\lambda^2 z}, \qquad (11.13)$$

which can be used to calculate the Lagrangian coordinates. According to (11.11), the following equalities are also true:

$$\frac{v_2}{v_{2_0}}=\left(\frac{r_2}{r_0}\right)^{-\nu/2}, \quad \frac{p_2}{p_{2_0}}=\left(\frac{r_2}{r_0}\right)^{-\nu}$$

where v_{2_0} and p_{2_0} are the velocity and pressure behind the shock wave front at the instant the shock passes through the point with the coordinate r_0.

The solution of the problem of a violent explosion is represented by using final formulas in parametric form with the variable parameter *V*. The range of variation of *V* and the character of the motion near the centre of symmetry depend on the relative location of the point *M* corresponding to the gas motion behind the shock front and on the singular point $\widetilde{E}$, which, as was explained in §5, belongs to the integral (11.9) and which appears in the $z > 0$ region for large γ (see Fig, 35).

According to (11.8) and (5.14), the condition that the values of V_2 at the point *M* and of V^* at the point $\widetilde{E}$ shall be equal, namely,

$$\frac{2}{2+\nu(\gamma-1)}=\frac{4}{(\gamma+1)(\nu+2)}$$

is attained for $\gamma = 7$ when $\nu = 3$; for $\gamma = \infty$ when $\nu = 2$ and for $\gamma = -1$ when $\nu = 1$.

Therefore, the range of variation of the variable V for $\nu = 1{,}2$ and $\gamma > 1$ is determined by the inequality

$$\frac{2}{(\nu+2)\gamma} \le V \le \frac{4}{(\nu+2)(\gamma+1)}. \qquad (11.14a)$$

The value $V = \dfrac{4}{(\nu+2)(\gamma+1)}$ in (11.14a) corresponds to the shock wave and the value $V = \dfrac{2}{(\nu+2)\gamma}$ corresponds to the centre of the explosion.

If $\nu = 3$ and $\gamma < 7$, then (11.14a) is also true; the range of variation of V for $\nu = 3$ and $\gamma > 7$ is determined by the inequalities:

$$\frac{4}{5(\gamma+1)} \le V \le \frac{2}{5}. \qquad (11.14b)$$

The value V = 2/5 in (11.14b) corresponds to the boundary of the expanding vacuum and $V = \dfrac{4}{5(\gamma+1)}$ corresponds to the shock wave. Hence, a vacuum is obtained only in the spherical case for finite large γ.

When the calculations described are carried out, the complete solution is given by the formulas:

$$\frac{r}{r_2} = \left[\frac{(\nu+2)(\gamma+1)}{4}V\right]^{-2/(2+\nu)} \left[\frac{\gamma+1}{\gamma-1}\left(\frac{(\nu+2)\gamma}{2}V-1\right)\right]^{-\alpha_2} \left[\frac{(\nu+2)(\gamma+1)}{(\nu+2)(\gamma+1)-2[2+\nu(\gamma-1)]}\left(1-\frac{2+\nu(\gamma-1)}{2}V\right)\right]^{-\alpha_1}$$

$$\frac{r_0}{r_2} = \left[\frac{(\nu+2)(\gamma+1)}{4}V\right]^{-2/(2+\nu)} \left[\frac{\gamma+1}{\gamma-1}\left(\frac{(\nu+2)\gamma}{2}V-1\right)\right]^{\alpha_4} \left[\frac{(\nu+2)(\gamma+1)}{(\nu+2)(\gamma+1)-2[2+\nu(\gamma-1)]}\left(1-\frac{2+\nu(\gamma-1)}{2}V\right)\right]^{\alpha_7} \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{\nu+2}{2}V\right)\right]^{-[\alpha_6+\alpha_7]-[2/(2+\nu)]}$$

$$\frac{v}{v_2} = f = \frac{(\nu+2)(\gamma+1)}{4}V\frac{r}{r_2},$$

$$\frac{\rho}{\rho_2} = g = \left[\frac{\gamma+1}{\gamma-1}\left(\frac{(\nu+2)\gamma}{2}V-1\right)\right]^{\alpha_3} \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{\nu+2}{2}V\right)\right]^{\alpha_5} \left[\frac{(\nu+2)(\gamma+1)}{(2+\nu)\gamma+1)-2[2+\nu(\gamma-1)}\left(1-\frac{2+\nu(\gamma-1)}{2}V\right)\right]^{\alpha_2}$$

$$\frac{p}{p_2} = h = \left[\frac{(\nu+2)(\gamma+1)}{4}V\right]^{2\nu/(2+\nu)} \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{\nu+2}{2}V\right)\right]^{\alpha_5+1} \left[\frac{(\nu+2)(\gamma+1)}{4}V\right]^{2\nu/(2\nu)} \left[\frac{\gamma+1}{\gamma-1}\left(1-\frac{\nu+2}{2}V\right)\right]^{\alpha_5+1}$$

$$\frac{T}{T_2} = \frac{p}{p_2}\frac{\rho_2}{\rho}$$

(11.15)

where

$$\alpha_1 = \frac{(\nu+2)\gamma}{2+\nu(\gamma-1)}\left[\frac{2\nu(2-\gamma)}{\gamma(\nu+2)^2}-\alpha_2\right], \qquad \alpha_2 = \frac{1-\gamma}{2(\gamma-1)+\nu},$$

$$\alpha_3 = \frac{\nu}{2(\gamma-1)+\nu}, \qquad \alpha_4 = \frac{\alpha_1(\nu+2)}{2-\gamma},$$

$$\alpha_5 = \frac{2}{\gamma-2}, \qquad \alpha_6 = \frac{\gamma}{2(\gamma-1)+\nu},$$

$$\alpha_7 = \frac{[2+\nu(\gamma-1)]\alpha_1}{\nu(2-\gamma)}.$$

(11.16)

It is easy to derive asymptotic formulas for v, ρ, p and the temperature T near the centre of the explosion from (11.15) for $\gamma < 7$ as $V \to \dfrac{2}{(2+\nu)\gamma}$ and $r \to 0$. We find

$$\left.\begin{aligned}
v &= \frac{2}{(2+\nu)\gamma}\frac{r}{t} \\
\rho &= k_1\rho_1\left(\frac{E}{\rho_1}\right)^{-\nu/(\nu+2)(\gamma-1)} t^{-2\nu/(\nu+2)(\gamma-1)} r^{\nu/(\gamma-1)} \\
p &= k_2\rho_1\left(\frac{E}{\rho_1}\right)^{2/(\nu+2)} t^{-2\nu/(\nu+2)} \\
T &= k_3\frac{1}{c_0}\left(\frac{E}{\rho_1}\right)^{[2(\gamma-1)+\nu]/(\nu+2)(\gamma-1)} t^{[2\nu(2-\gamma)]/(\nu+2)(\gamma-1)} r^{-\nu/(\gamma-1)}
\end{aligned}\right\} \tag{11.17}$$

where for **spherical** symmetry,

$$\begin{aligned}
k_1 &= \frac{(\gamma+1)^{[3\gamma(\gamma+1)]/(\gamma-1)(3\gamma-1)}}{2^{6/[5(\gamma-1)]}\gamma^{(7\gamma-1)/(\gamma-1)(3\gamma-1)}}(\gamma-1)^{-1}\left(\frac{2\gamma+1}{7-\gamma}\right)^{(13\gamma^2-7\gamma+12)/[5(\gamma-1)(2-\gamma)(3\gamma-1)]} \\
k_2 &= \frac{0.32}{2^{6/5}}\frac{(\gamma+1)^{(\gamma+1)/(3\gamma-1)}}{\gamma^{(4\gamma)/(3\gamma-1)}}\left(\frac{2\gamma+1}{7-\gamma}\right)^{(13\gamma^2-7\gamma+12)/[5(2-\gamma)(3\gamma-1)]} \\
k_3 &= \frac{k_2}{k_1(\gamma-1)}
\end{aligned} \tag{11.18}$$

for **cylindrical** symmetry,

$$\left.\begin{aligned}
k_1 &= \frac{(\gamma+1)^{(\gamma+1)/(\gamma-1)}}{\gamma-1}\frac{\gamma^{(3\gamma-4)/(\gamma-1)(2-\gamma)}}{2^{2/(\gamma-1)(2-\gamma)}} \\
k_2 &= \frac{\gamma^{[2(\gamma-1)]/(2-\gamma)}}{2^{(4-\gamma)/(2-\gamma)}} \\
k_3 &= \frac{k_2}{k_1(\gamma-1)}
\end{aligned}\right\} \tag{11.19}$$

for **plane** waves,

$$\left.\begin{aligned}
k_1 &= \frac{(2\gamma-1)^{(5\gamma-4).[3(\gamma-1)(2-\gamma)]}(\gamma+1)^{(4+\gamma-3\gamma^2)/[3(\gamma-1)(2-\gamma)]}}{2^{2/[3(\gamma-1)]}\gamma^{1/(\gamma-1)}(\gamma-1)} \\
k_2 &= \frac{2^{7/3}}{9}\frac{(2\gamma-1)^{(5\gamma-4)/[3(2-\gamma)]}}{(\gamma+1)^{[2(\gamma+1)]/[3(2-\gamma)]}} \\
k_3 &= \frac{k_2}{k_1(\gamma-1)}
\end{aligned}\right\}. \tag{11.10}$$

The variation of the constants k_1, k_2 and k_3 with γ is shown in Fig. 63.

The velocity is close to zero near the centre of symmetry; the pressure is not zero and is asymptotically constant in the r coordinate; the density approaches zero very rapidly and the temperature approaches infinity. It is easy to see that the entropy also approaches infinity. Large temperature gradients occur near the centre of the explosion; consequently, the heat conduction property becomes very important. If heat conduction is taken into account, then the temperature is finite at the centre of the explosion.

The mass of gas disperses from the centre of the explosion (we have $\rho \approx 0$ for $r = 0$), the pressure is finite at the centre but approaches zero as time passes.

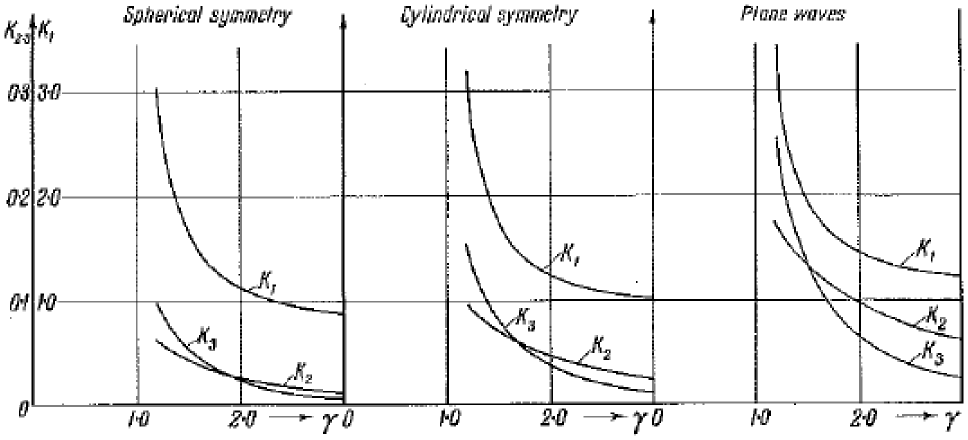

FIG. 63. The quantities $k_1(\gamma)$, $k_2(\gamma)$, $k_3(\gamma)$ in (11.17) which give asymptotic values of the density, pressure and temperature near the centre of the explosion.

Hence, it is clear that a **reverse gas motion** towards the centre of the explosion must occur in a violent explosion in a gas in which there is finite pressure prior to the explosion. We observe this effect well in such explosions in which repeated **pulsations of the gas bubble** arise.

An **Archimedean lift**, which causes a disturbance in the ambient atmosphere, results from the outward rush of gas from the centre and the reduction in pressure. During the atomic bomb explosion in New Mexico, the vertical lifting velocity of the luminous core was of the order of 35 m/sec. according to data on the photographs.

Graphs of the velocity, density, pressure and temperature distribution behind the wave front are given in Figs. 64 - 71: they define the motion and the changes of state in the gas in the three cases, $\nu = 1, 2, 3$.

Certain numerical values in practical applications are given in the tables.

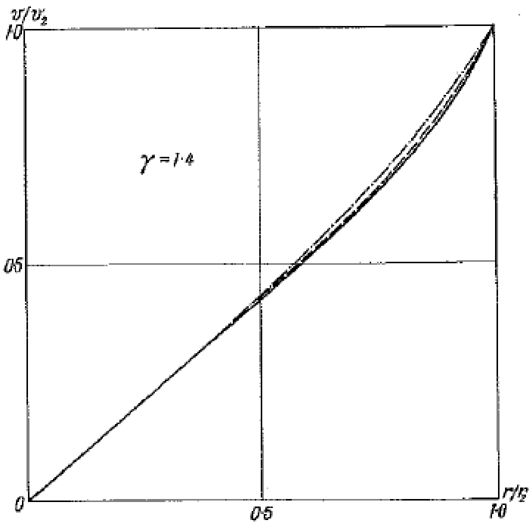

FIG. 64. Velocity distribution behind the shock wave ——— spherical case; – – – – – cylindrical case; –·–·–·–· plane case.

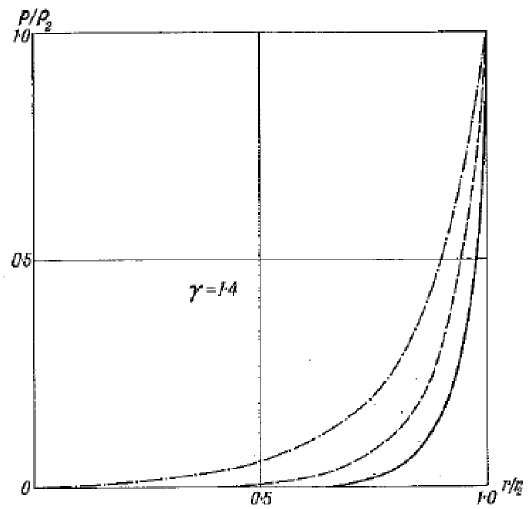

FIG. 65. Density distribution behind the shock wave.

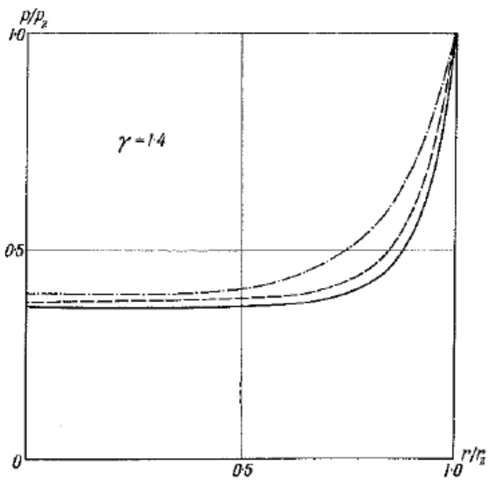

FIG. 66. Pressure distribution behind the shock wave ——— spherical case; ------ cylindrical case; –·–·–·–· plane case.

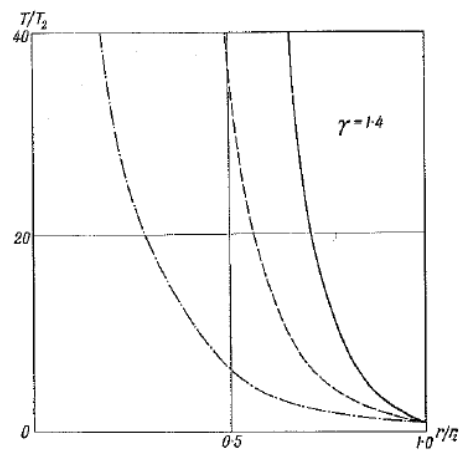

FIG. 67. Temperature distribution behind the shock wave.

The influence of the specific heat ratio γ on the distribution of the physical variables behind the shock front are shown in Figs. 72, 73, and 74 for spherical symmetry.

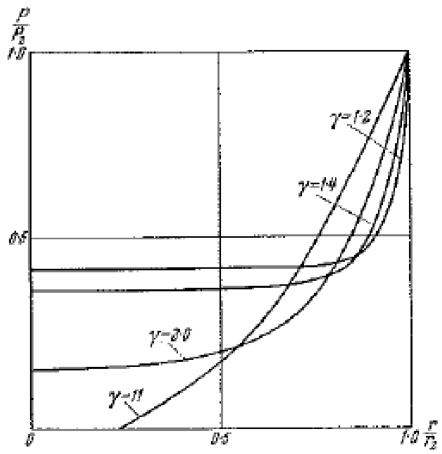

FIG. 72. Influence of the constant γ on the pressure distribution behind the shock wave front for spherical symmetry.

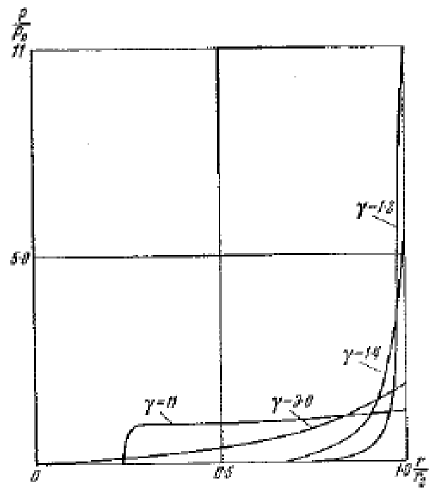

FIG. 73. Influence of the constant γ on the density distribution behind the shock wave front for spherical symmetry.

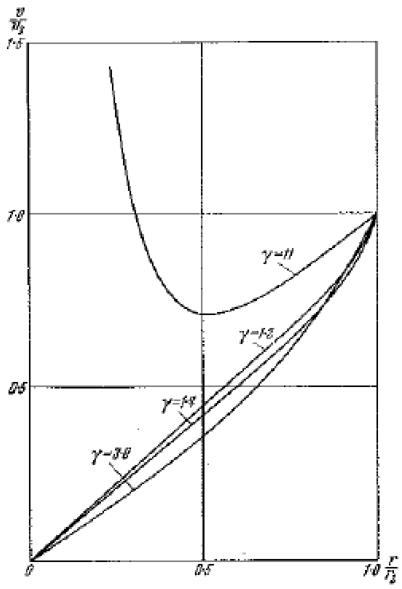

FIG. 74. Influence of the constant γ on the velocity distribution behind the shock wave front for spherical symmetry.

We note that the solution is particularly simple for $\gamma = 7$. We have

$$\frac{v}{v_2} = \frac{r}{r_2} = \lambda\,; \quad \frac{\rho}{\rho_2} = \lambda\,; \quad \frac{p}{p_2} = \lambda^3. \qquad (11.21)$$

In this case, we find

$$v = \rho = p = 0$$

at the centre of symmetry.

If $\gamma > 7$, then an empty sphere of radius r^* is produced at the centre; the

pressure is zero within this sphere. The radius r^* increases with time so that the ratio r^*/r_2, which depends only on γ remains constant.

The high values of the index γ arise when a violent explosion is initiated in a compressible medium such as water.

The problem of a point explosion formulated above is solved for any constant value of the abstract parameter γ. If $\gamma = \gamma^* = c_p / c_v$, then the process is adiabatic for each gas particle. If the constant γ is arbitrary and not equal to $\gamma^* = c_p / c_v$, then the process will be poly tropic; in this case the jump front is similar to a detonation front or to a front of phase transition with, heat absorption.

Actually, condition (2.6) can be rewritten as:

$$\frac{\gamma^* p_1}{(\gamma^*-1)\rho_1} + \frac{1}{2}(v_1 - c)^2 + Q = \frac{\gamma^* p_2}{(\gamma^*-1)\rho_2} + \frac{1}{2}(v_2 - c)^2 , \tag{11.22}$$

where

$$Q = \frac{\gamma - \gamma^*}{(\gamma-1)(\gamma^*-1)}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right);$$

Since $\frac{p_2}{\rho_2} = \boldsymbol{R}T_2 > \boldsymbol{R}T_1 = \frac{p_1}{\rho_1}$, then evidently, condition (11.22) is analogous to the condition on a detonation front for $\gamma > \gamma^* = c_p / c_v > 1$ when $Q > 0$. Heat absorption ($Q < 0$) will occur on the front if $\gamma > 1$ and $\gamma < \gamma^*$.

On the other hand, the following equation for a reversible process holds on each particle in the flow behind the jump front

$$dQ = TdS = Tc_v d \ln \frac{p}{\rho^{\gamma^*}} = Tc_v d \ln \frac{p}{\rho^{\gamma}} \rho^{\gamma-\gamma^*} ;$$

from which, since $\frac{p}{\rho^{\gamma}} = \text{const.}$ and $p = \boldsymbol{R}\rho T$ on each particle, we obtain:

$$dQ = Tc_v(\gamma - \gamma^*)\frac{d\rho}{\rho} = \frac{\gamma - \gamma^*}{\gamma - 1} c_v dT . \tag{11.23}$$

We would have $d\rho < 0$ and $dT < 0$ on fixed particles in the solutions considered; consequently, we have a process with heat release per particle $dQ > 0$ for $\gamma < \gamma^*$ and with heat absorption $dQ < 0$ for $\gamma > \gamma^*$.

The motion of a gas containing very fine solid or liquid particles (**dusty atmosphere**) can be considered. It can be shown (Sidorkina, 1957) that the adiabatic equations of motion and the conditions at jumps in such a mixture are in agreement with the equations for a gas without impurities in which the specific heat is constant on solid particles, but with the value of the parameter $\gamma < \gamma^* = c_p / c_v$ varying, where c_p and c_v are the appropriate specific heats for the gas without the impurities. The presence of impurities on the jump front leads to heat absorption $Q < 0$; in the stream, the heated particles give off heat to the gas so that $dQ > 0$.

The constant E, which must be expressed in terms of the charge energy E_0 (equal to the total energy of the disturbed gas in the present formulation of the problem), enters into the formulas obtained above giving the dependent variables of the dimensional motion. The nondimensional variables of the motion are represented by standard curves independent of the explosion energy E_0 or of the proportional quantity E.

We have the following formulas for the total energy:

$$E_0 = \int_0^{r_2} \frac{\rho v^2}{2} 4\pi r^2 dr + \int_0^{r_2} \frac{p}{\gamma - 1} 4\pi r^2 dr\,,$$ in the **spherical** case;

$$E_0 = \int_0^{r_2} \frac{\rho v^2}{2} 2\pi r dr + \int_0^{r_2} \frac{p}{\gamma - 1} 2\pi r dr\,,$$ in the **cylindrical** ease;

$$\frac{1}{2} E_0 = \int_0^{r_2} \frac{\rho v^2}{2} dr + \int_0^{r_2} \frac{p}{\gamma - 1} dr\,,$$ for **plane** waves.

The first term is the **kinetic** and the second is the **thermal gas energy**. Introducing nondimensional quantities, we find

$$E_0 = \alpha(\gamma) E\,, \qquad (11.24)$$

where

$$\alpha = 2\pi \int_0^1 R V^2 \lambda^4 d\lambda + \frac{4\pi}{\gamma - 1} \int_0^1 P \lambda^4 d\lambda\,,$$ in the **spherical** case;

$$\alpha = \pi \int_0^1 R V^2 \lambda^3 d\lambda + \frac{2\pi}{\gamma - 1} \int_0^1 P \lambda^3 d\lambda\,,$$ in the **cylindrical** case;

$$\alpha = \int_0^1 R V^2 \lambda^2 d\lambda + \frac{2}{\gamma - 1} \int_0^1 P \lambda^2 d\lambda$$ for **plane** waves.

The function $\alpha(\gamma)$ is shown in Fig. 75, calculated for spherical, cylindrical and planar symmetry.

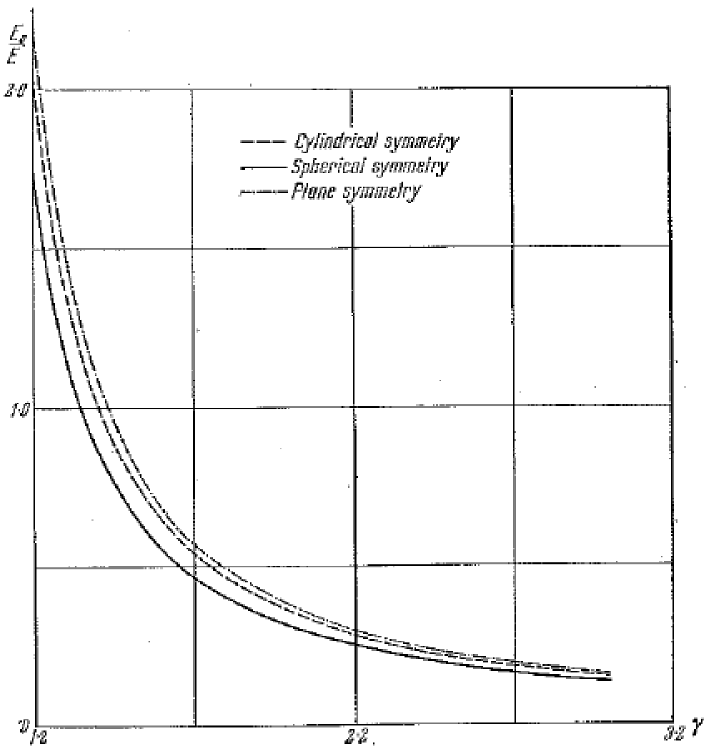

FIG. 75. The ratio $E_0/E = \alpha$ as a function of γ.

In particular, we have for $\gamma = 1.4$, in the case of spherical symmetry.

$$E_0 (0.186 + 0.665) E = 0.851 E\,,$$

or

$$E = 1.175 E_0\,.$$

If we use the value of E given in (11.7) which was obtained from

experimental results for the atomic bomb explosion in New Mexico, and if we take $\gamma = 1.4$, then we obtain $E_0 = 7.19 \times 10^{20}$ erg for the energy of the explosion,[3] which corresponds to the energy liberated by exploding 16,800 tons of TNT.

2. ON TAKING HEAT CONDUCTION INTO ACCOUNT

We noted above that the properties of viscosity and heat conduction can exert a definite influence on the gas motion near the centre of an explosion; if viscosity and heat conduction are not neglected, then the coefficient of viscosity μ and the coefficient of heat conduction κ enter into the equations of motion.

It should he noted that the coefficient of heat conduction enters as a factor of the temperature *T*.

When the temperature is eliminated by using the equation of state (for simplicity, we assume that it is the Clapeyron equation $T = \dfrac{p}{\boldsymbol{R}\rho}$), the ratio $\kappa / \boldsymbol{R}$ ($\boldsymbol{R}$ is the gas constant), rather than κ itself, will actually enter as a coefficient. As is known, the coefficients μ and κ depend on the temperature. Usually, they are assumed to be proportional to *T* to some power (most often, proportional to $T^{1/2}$ or a constant). Let us assume that

$$\mu = \mu_1 T^{\alpha} \quad \text{and} \quad \kappa = \kappa_1 T^{\alpha}$$

where μ_1 and κ_1 are constants.

After eliminating *T*, the new dimensional constants

$$\frac{\mu_1}{R^{\alpha}} \quad \text{and} \quad \frac{\kappa_1}{R^{\alpha+1}}$$

enter into the equations.

Their dimensions are

$$\left[\frac{\mu_1}{R^{\alpha}}\right] = \left[\frac{\kappa_1}{R^{\alpha+1}}\right] = ML^{-1-2\alpha}T^{-1+2\alpha} .$$

In order that the motion may be self-similar, it is sufficient that these dimensions be expressed in terms of the dimensions of ρ_1, and E_0. It is easy to see that it is necessary to put $\alpha = 1/6$ in the **spherical** case,[4] $\alpha = 0$ in the **cylindrical** case and $\alpha = -1/2$ in the **plane** case.

Hence, the problem of an intense explosion can be solved by integration of ordinary differential equations taking viscosity and heat conduction into account, if it is assumed that

$\mu = \mu_1 T^{1/6}$, $\kappa = \kappa_1 T^{1/6}$ for **spherical** waves; (11.25)

$\mu = \text{const}$, $\kappa = \text{const}$ for **cylindrical** waves; (11.26)

$\mu = \dfrac{\mu_1}{\sqrt{T}}$, $\kappa = \dfrac{\kappa_1}{\sqrt{T}}$ for **plane** waves. (11.27)

Approximate solutions of the problem of an intense point explosion taking heat conduction into account have been published recently (Korobeinikov, 1957).

It is not difficult to see that self-similarity of the problem of an intense explosion is retained for a perfect gas and for many other media in which the temperature is uniform in space, but varies with time in the disturbed region

[3] In estimating the total energy liberated in an atomic bomb explosion, it should be kept in mind that a considerable part of the energy is expended in radiation.

[4] This result is obtained in Bam-Zelikovich (1949). We have $\left[\dfrac{\mu_1}{R^{\alpha}}\right] = \left[\left(E\rho_1{}^2\right)^{1/3}\right]$ for $\alpha = 1/6$.

as a result of the intense heat exchange (due to very large heat conduction or to radiation and other processes).

These conditions apply when the adiabatic equation can be replaced by

$$\frac{\partial T}{\partial r} = 0 .$$

The solution of the appropriate problems for spherical waves has been published by Korobeinikov (1956a).

3. SELF-SIMILARITY OF A POINT EXPLOSION IN IDEAL MEDIA

A further remark on solving the problem of an intense explosion by ideal fluid theory, with a more general kind of equation of state and an expression of the internal gas energy as a function of p and ρ, is appropriate.[5] The internal energy function $\varepsilon(p,\rho)$ enters directly into the shock wave conditions and into the heat flow equation. In the general case, it can always be represented as

$$\varepsilon = \frac{p}{\rho_1}\phi\left(\frac{p}{p^*},\frac{\rho}{\rho_1}\right) + \text{const} ,$$

where ϕ is a nondimensional function of its arguments and p^* is any constant with the dimensions of pressure.

Since the dimensions of p^* cannot be expressed in terms of the dimensions of ρ_1 and E, then it is sufficient, for the motion to be self-similar, that ε should not contain p^*, i.e.,

$$\varepsilon = \frac{p}{\rho_1}\phi\left(\frac{\rho}{\rho_1}\right), \tag{11.28}$$

where ϕ is an arbitrary function of its argument.

No new physical dimensional constants can appear in the adiabatic equation

$$d\varepsilon + pd\frac{1}{\rho} = 0 .$$

Condition (11.28) imposes a certain limitation on the equation of state. Actually, since

$$\frac{d\varepsilon + p\frac{1}{\rho}}{T} = dS$$

is a total differential, T and ε must satisfy the following partial differential equation

$$T + \frac{\partial T}{\partial p}\left(\rho^2\frac{\partial \varepsilon}{\partial \rho} - p\right) - \rho^2\frac{\partial T}{\partial \rho}\frac{\partial \varepsilon}{\partial p} = 0 .$$

Substituting $\varepsilon = p\phi(\rho)$ (for simplicity, we can put $\rho_1 = 1$), we obtain:

$$T + \frac{\partial T}{\partial p}p(\phi'(\rho)\rho^2 - 1) - \rho^2\phi(\rho)\frac{\partial T}{\partial \rho} = 0 .$$

An equivalent system of ordinary differential equations can be written:

$$\frac{dT}{T} = -\frac{dp}{p[\phi'(\rho)\rho^2 - 1]} = \frac{d\rho}{\phi(\rho)\rho^2} .$$

It has two integrals

$$T\exp\left(-\int\frac{d\rho}{\rho^2\phi(\rho)}\right) = C_1 \quad\text{and}\quad p\phi(\rho)\exp\left(-\int\frac{d\rho}{\rho^2\phi(\rho)}\right) = C_2 ,$$

[5] See Bam-Zelikovich (1949).

therefore,

$$p\phi(\rho)\exp\left(-\int \frac{d\rho}{\rho^2\phi(\rho)}\right) = \Phi\left[T\exp\left(-\int \frac{d\rho}{\rho^2\phi(\rho)}\right)\right],$$

(11.29)

where ϕ and Φ are arbitrary functions of their arguments. Equation (11.29} is satisfied by all equations of state in which the temperature is proportional to the pressure and depends on the density arbitrarily. However, in spite of the presence of two arbitrary functions, many interesting equations of state (for example, the van der Waal) cannot be written in the form (11.29),

If the function Φ reduces to a constant, the pressure and, therefore, the internal energy depend only on the density; a corresponding dimensional constant, equal to Φ, enters into the internal energy expression. In this case, the system depends on one parameter; consequently, the preceding formulation of the problem becomes impossible.

The function Φ in (11.29) in the general case depend on a number of dimensional constants which cannot violate the self-similarity of the problem formulated in terms of v, p, ρ in §1, However, the presence of these constants can lead to a non-self-similar relation between the temperature T and r and t in the disturbed flow of the medium.

We note that (11.28) gives

$$p\phi(\rho)\exp\left(-\int \frac{d\rho}{\rho^2\phi(\rho)}\right) = \psi(S) \quad \text{and} \quad T\exp\left(-\int \frac{d\rho}{\rho^2\phi(\rho)}\right) = \psi'(S),$$

where $\psi(S)$ is a function of the entropy which depends on the type of function Φ in (11.29) and which satisfies the equation

$$\psi(S) = \Phi(\psi'(S)).$$

4. POINT EXPLOSION IN AN IDEAL INCOMPRESSIBLE FLUID

The problem of the point explosion can be analysed under the assumption that the medium is **incompressible**. The adiabatic equation can be replaced by

$$\rho = \rho_1 = \text{const.} \qquad (11.30)$$

The disturbance is propagated with an infinitely high velocity in this case; consequently, a solution without a shock wave is possible. The case of incompressible fluid motion during a point explosion can be obtained as the limit of adiabatic gas motion as $\gamma \to +\infty$. As in the general case, if $p_1 = 0$, the fluid motion is self-similar and, as is easily verified, the following formula is true for the velocity field:

$$v = \frac{2}{5}\left(\frac{E}{2\pi\rho_1}\right)^{3/5}\frac{t^{1/5}}{r^2}, \qquad (11.31)$$

since the motion must correspond to a source of variable intensity dependent only on E, ρ_1, t. The constant factor is determined by assigning the fluid kinetic energy which equals $\frac{4}{25}E$. It follows from the Lagrange integral that

$$\frac{p}{\rho_1} = \frac{2}{25}\left(\frac{E}{2\pi\rho_1}\right)^{2/5} t^{-6/5}\frac{r^*}{r}\left[1-\left(\frac{r^*}{r}\right)^3\right]. \qquad (11.32)$$

A spherical vacuum with, increasing radius r^* is formed at the centre, for

which we find

$$r^* = \left(\frac{E}{2\pi\rho_1}\right)^{1/5} t^{2/5}. \tag{11.33}$$

The pressure is zero inside a sphere of radius r^*.

Curve 1 on Fig. 76 gives the pressure distribution in the fluid which is a universal curve in the variables

$$\frac{pt^{6/5}}{\rho_1\left(\frac{E}{2\pi\rho_1}\right)^{2/5}};\quad \frac{r}{r^*}.$$

The peak pressure drops in inverse proportion to $t^{6/5}$, while the peak pressure is reached independently of the time at $\frac{r^*}{r_{p,\max}} = 4^{-1/3}$, from which

$$r_{p,\max} = 4^{1/3} r^* = 4^{1/3}\left(\frac{E}{2\pi\rho_1}\right)^{1/5} t^{2/5}, \tag{11.34}$$

Only the ratio E/ρ_1 is essential in the motion found.

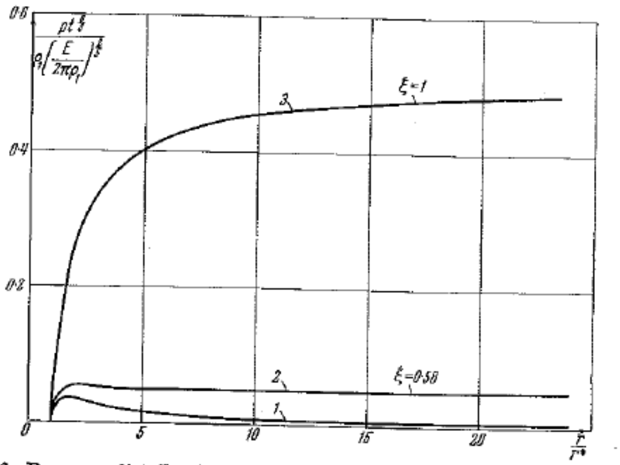

FIG. 76. Pressure distribution in an incompressible fluid for a point explosion. 1, self-similar solution; 2, solution taking counter-pressure into account for small $\xi = r^*/r^*_{\max}$; 3, pressure when the internal cavity has the maximum radius.

If $p_1 \neq 0$, the motion of an incompressible fluid, is not self-similar, just as for a gas. However, the complete solution is easily obtained in simple analytic form in this case.

The following formulas can be written for the velocity potential and for the magnitude of the velocity:

$$\phi = -\frac{r^{*2}\frac{dr^*}{dt}}{r} \quad \text{and} \quad v = \frac{r^{*2}}{r^2}\frac{dr^*}{dt}, \tag{11.35}$$

which are true for any law $r^*(t)$ according to which the internal cavity expands.

We obtain from the Lagrange integral, if the internal pressure is $p^* = 0$ and the external pressure at infinity is $p_1 = \text{const.}$:

$$-\frac{p_1}{\rho} = r^*\frac{d^2r^*}{dt^2} + \frac{3}{2}\left(\frac{dr^*}{dt}\right)^2. \tag{11.36}$$

Hence

$$\left(\frac{dr^*}{dt}\right)^2 = -\frac{2}{3}\frac{p_1}{\rho_1} + Cr^{*-3}. \qquad (11.37)$$

We find from the solution of the self-similar problem using (11.33):

$$C = \frac{4}{25}\left(\frac{E}{2\pi\rho_1}\right).$$

If the following notation is introduced

$$\tau = t\frac{(25\pi)^{1/3}2^{1/2}p_1{}^{5/6}}{3^{5/6}\rho_1{}^{1/2}E^{1/3}}, \qquad (11.38)$$

$$\xi = \frac{r^*}{r^*_{\max}}, \quad r^*_{\max} = \left(\frac{3}{25\pi}\frac{E}{p_1}\right)^{1/3},$$

then the rate of expansion of the internal cavity is given by the universal formula

$$\tau = \int_0^{\xi}\frac{d\xi}{\sqrt{\frac{1}{\xi^3}-1}}. \qquad (11.39)$$

It is evident that $r_{\max}$ will equal the greatest radius of the internal cavity.

The expansion, law given by (11.39) is shown on the graph (Fig. 77). The expansion law for self-similar motion in the same variables is given by

$$\tau = \frac{2}{5}\xi^{5/2}; \qquad (11.40)$$

corresponding to the dashed curve on Fig. 77. Also superposed on Fig. 77 is the curve for the self-similar solution for a gas with $\gamma = 11$.

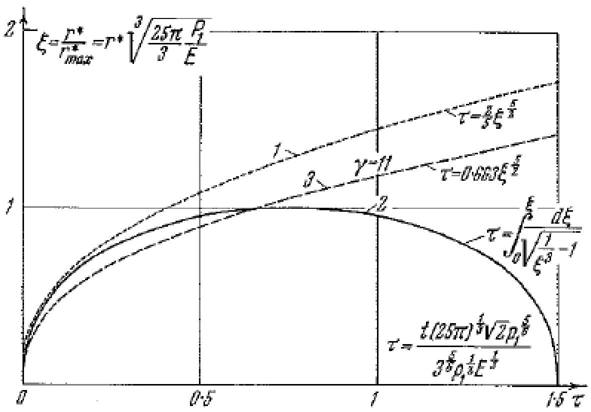

FIG. 77. Expansion law of the internal cavity; 1, self-similar solution; 2, incompressible fluid motion taking counter-pressure into account; 3, compressible medium with $\gamma = 11$.