

Michael Spivak
A Comprehensive Introduction to Differential Geometry
Volume One, Third Edition, 1999.
Table of Contents

CHAPTER 1. MANIFOLDS

Elementary properties of manifolds

Examples of manifolds

Problems

CHAPTER 2. DIFFERENTIAL STRUCTURES

C^∞ structures

C^∞ functions

Partial derivatives

Critical points

Immersion theorems

Partitions of unity

Problems

CHAPTER 3. THE TANGENT BUNDLE

The tangent space of \mathbb{R}^n

The tangent space of an imbedded manifold

Vector bundles

The tangent bundle of a manifold

Equivalence classes of curves and derivations

Vector fields

Orientation

Addendum. Equivalence of Tangent Bundles

Problems

CHAPTER 4. TENSORS

The dual bundle

The differential of a function

Classical *versus* modern terminology

Multilinear functions

Covariant and contravariant tensors

Mixed tensors and contraction

Problems

CHAPTER 5. VECTOR FIELDS AND DIFFERENTIAL EQUATIONS

Integral curves

Existence and uniqueness theorems

The local flow

One-parameter groups of diffeomorphisms

Lie derivatives

Brackets

Addendum 1. Differential Equations

Addendum 2. Parameter Curves in Two Dimensions

Problems

CHAPTER 6. INTEGRAL MANIFOLDS

Prologue; classical integrability theorems

Local Theory; Frobenius integrability theorem

Global Theory

Problems

CHAPTER 7. DIFFERENTIAL FORMS

Alternating functions

The wedge product

Forms

Differential of a form

Frobenius integrability theorem (second version)

Closed and exact forms

The Poincaré Lemma

Problems

CHAPTER 8. INTEGRATION

Classical line and surface integrals

Integrals over singular k -cubes

The boundary of a chain

Stokes' Theorem

Integrals over manifolds

Volume elements

Stokes' Theorem

de Rham cohomology

Problems

CHAPTER 9. RIEMANNIAN METRICS

Inner products

Riemannian metrics

Length of curves

The calculus of variations

The First Variation Formula and geodesics

The exponential map

Geodesic completeness

Addendum. Tubular Neighborhoods

Problems

CHAPTER 10. LIE GROUPS

Lie groups

Left invariant vector fields

Lie algebras

Subgroups and subalgebras

Homomorphisms

One-parameter subgroups

The exponential map

Closed subgroups

Left invariant forms

Bi-invariant metrics

The equations of structure

Problems

CHAPTER 11. EXCURSION IN THE REALM OF ALGEBRAIC TOPOLOGY

Complexes and exact sequences

The Mayer-Vietoris sequence

Triangulations

The Euler characteristic

Mayer-Vietoris sequence for compact supports

The exact sequence of a pair

Poincaré Duality

The Thom class

Index of a vector field

Poincaré-Hopf Theorem

Problems

APPENDIX A

To Chapter 1

Problems

To Chapter 2

Problems

To Chapter 6

To Chapters 7, 9, 10

Problem

NOTATION INDEX .

INDEX