The Air Pressure on a Cone Moving at High Speeds. - II.

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§ 1. Comparison with Exprimemal Results.

Though the calculations are concerned only with the flow in the neighbourhood of an infinite cone, it was thought that the flow near a finite cone or near the nose of a bullet with a conical head might be comparable with the calculations.

Accordingly, measurements of two kinds were undertaken. The surface pressures on cones were measured at the National Physical Laboratory, and photographs of a bullet with a 60° conical head were taken at various speeds by the Research Department at Woolwich.

§ 2. Comparison between the Critical Speeds of Cone and Wedge.

The critical speeds at which the shock wave leaves the edge of a wedge can be found directly from the curves of fig. 7, Part I. For a wedge of angle $2(\beta-\alpha)$ moving along the bisector of its faces all possible values of y and the corresponding values of α are given by the intersections of the line $\beta-\alpha = const$ with the successive contours of y. Since U/a is the single-valued function of y given by equation (10B) all the points on the diagram of fig. 7 which correspond with the critical condition of minimum velocity for a given wedge or wedge of maximum angle for a given speed lie on the line which passes through the maximum values of $\beta-\alpha$, on each y contour. This line is shown dotted in fig. 7 and the corresponding values of $\beta-\alpha$, y and U/a are given in Table IV.

Table IV.

y.	0.	0·0005.	0·006.	0·02.	0·05.	0·10.	0·15.
θ_s or $\beta-\alpha$ U/a	45·4 ∞	42·5 6·26	38·8 4·08	35·3 3·21	30·8 2·60	25·4 2·16	21·0 1·90

y.	0·20.	0·25.	0·30.	0·35.	0·40.	0·45.	0·50.
θ or $\beta-\alpha$ U/a	17·1 1·71	13·6 1·56	10·3 1·43	7·2 1·32	4·5 1·22	2·2 1·13	0·4 1·045

These values are plotted in fig. 3 which shows the relationship between

U/a and θ_s or $\beta - \alpha$ for a wedge. The corresponding points for a cone, namely ($\theta_s = 10°$, U/a = 1.035), ($\theta_s = 20°$, U/a = 1.18), ($\theta_s = 30°$, U/a = 1.46), are also marked in fig. 3 and joined by a curve.

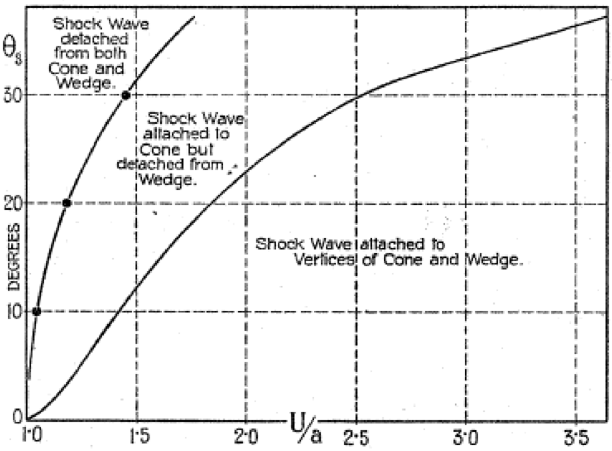

FIG. 3.

It will be seen that as the speed is reduced from high values the shock wave leaves the edge and travels ahead of a wedge long before it leaves the point of a cone having the same vertex angle.

§ 3. Comparison with Approximate Theory of v. Karman and Moore.

In a recent paper v. Karman and Moore *(loc. cit.,* Part I) have given an approximate theory of the disturbances produced by a thin spindle-shaped body moving at speeds greater than that of sound. As an example of their general method they give the approximate solution for a cone of small angle. The motion which their solution presupposes is an irrotational one without shock waves. It is evident from the foregoing work that a shock wave necessarily exists in all cases where the conical regime exists, for our method would find solutions without shock waves if they could exist. (In that case we would merely find that at the Mach cone $x = y$ and $z = 1$ would satisfy the required conditions.)

When the angle of the cone is small it may occur that the change in pressure at the shock wave is small compared with the change in pressure between the shock wave and the solid surface. Denoting the ratio "pressure change in shock wave" to "total pressure change from undisturbed air to solid surface" as

$$\frac{p_2 - p_1}{p_s - p_1}$$

by F, v. Karman and Moore's approximation might be expected to be valid if F is small compared with 1.0. The true value of F may be calculated in any given case from the figures given in Table II thus

$$\frac{p_2 - p_1}{p_1} = \frac{x}{y} - 1, \quad \frac{p_s - p_1}{p_1} = \left(\frac{p_s - p_1}{\rho_1 U^2}\right)\left(\gamma \frac{U^2}{a^2}\right)$$

and the values of each factor in F are given in the table. In the case $\theta_s = 10°$ the values of F are given for various values of U/a in the following table.

Table V.—$\theta_s = 10°$.

U/a	1·09	1·04	1·07	1·22	1·39	1·81	2·39	3·33	5·46
F	0·69	0·37	0·12	0·13	0·17	0·26	0·38	0·58	0·84

It will be seen that from U/a = 1.07 to 1.39, F is less than one-sixth, so that only one-sixth of the total change in pressure occurs in the shock wave, the remaining five-sixths occurring in the region of irrotational flow behind it. Outside this limited region F rapidly increases, so that at U = 1.81a one-quarter of the total pressure change occurs in the shock wave. So far as the ***distribution*** of pressure is concerned, v. Karman's approximation can therefore only be said to give a good representation in a limited range extending approximately from U/a = 1.07 to 1.39.

A similar conclusion might have been reached in a qualitative manner by inspection of fig. 13 or Table II, Part I. In that table it will be seen that the wave angle θ_w is within 1° of the Mach angle M over the range U/a = 1.07 to U/a = 1.81. At speeds greater than 1.81a the difference between them is greater than l°. The difference between θ_w and M may be regarded as a rough measure of the intensity of the shock wave, so that in the case of the 20° cone the shock wave is weak in the range U/a = 1.07 to 1.81.

In the case of the 40° cone θ_w does not differ from M by less than 7.0° in any part of the range, while in the case of the 60° cone the least value of this difference is 18.1°. Thus we may anticipate that the shock wave produced by the 40° cone is of considerable intensity at all speeds, and that that produced by the 60° cone is still more intense.

Though v. Karman and Moore's theory involves no sudden change in velocity or pressure at a shock wave, yet it does involve infinite values of the rates of change of these quantities at the cone whose semi-vertical angle is the Mach angle. The approximate solution might therefore still be a good approximation even in cases like that of the 20° cone at U/a = 1. 81 when the increase in pressure at the shock wave is a considerable fraction of the total pressure increase at the surface of the solid cone. A proper basis for comparison is to be found in the pressure at the surface. v. Karman and Moore give an expression for the pressure at any point in equation (27) of

their paper. Transferring this formula into the notation adopted in the present investigation and applying it to give p_s we find

$$\frac{p_s - p_1}{\rho_1 U^2} = \frac{1}{2}\left(1 - \frac{u_s{}^2}{U^2}\right)\left\{1 + \frac{U^2}{4a^2}\left(1 - \frac{u_s{}^2}{U^2}\right)\right\},$$

where

$$\frac{u_s}{U} = \frac{\sec\theta_s \sqrt{\cot^2\theta_s - \cot^2 M}}{\sqrt{cor^2\theta_s - \cot^2 M} + \tan\theta_s \cosh^{-1}(\cot\theta / \cot M)}$$

and

$$\sin M = \frac{a}{U}.$$

Using these formulae $\frac{p_s - p_1}{\rho_1 U^2}$ was calculated for θ_s = 10°, 20°, and 30°. These approximate values are shown as **broken line curves** in fig. 4 and the corresponding values obtained by **our complete solution** are also shown for comparison. It will be seen that in the case of the 20° cone (θ_s = 10°) the approximate solution agrees very well with the complete solution over the range U = 1.05*a* to U = 2.5*a*. In the case of the 40° cone the approximation is rather **badly wrong**, the least error being about 25 per cent. In the case of the 60° cone the approximate values are still further from the true values.

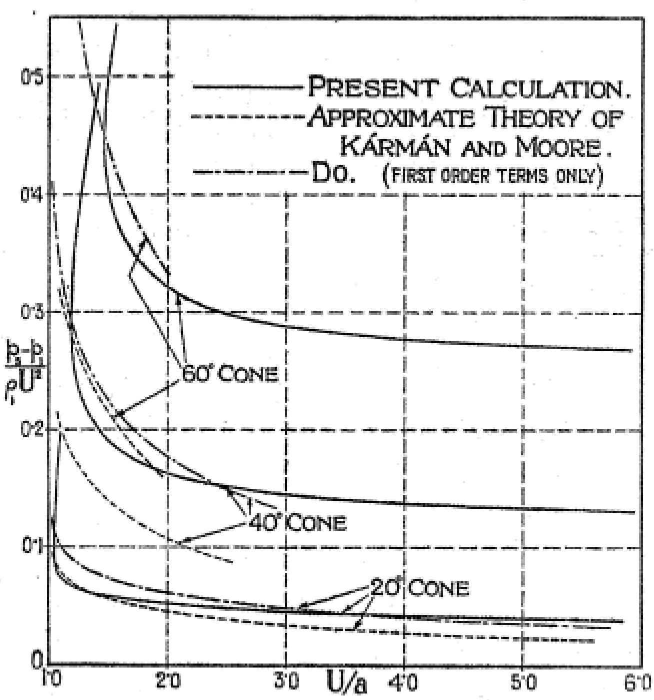

FIG. 4.

§ 4.

We are indebted to the Ordnance Committee for arranging for the photographic and wind channel investigations undertaken to test the theory and for assistance in computation. Our thanks are due in particular to Dr. A. D. Crow and Mr. T. Harris, of the Research Department, Woolwich, for carrying out the photographic work, and to Dr. H. J. Gough and Mr. A. Bailey, of the National Physical Laboratory, for carrying out the wind channel work. We are also indebted to Mr. W. Stott for computing the functions from which figs. 6, 7 and 8, Part I, were prepared.

[Note added December 18, 1932.]

Since writing this paper we have sent the diagram of fig. 4 to Professor v. Karman, who then reconsidered his theory in the light of the present calculations. He finds that his method of working out the second order terms is not quite logical, some of them having been neglected while others were retained. If only the first order terms are retained the agreement between our results and his is much improved. He has kindly calculated for us the first approximation to the values of $\dfrac{p_s - p_1}{\rho_1 U^2}$ for our three cones and his results are shown by a **third set of curves** in fig. 4. The fact that the retention of some, but not all, of the second order terms makes such a large difference to the results (as may be seen in fig. 4) shows that v. Karman and Moore's first order approximation **cannot be regarded as mathematically justifiable** outside the range where the first and second order approximations nearly coincide. This corresponds with the range where F (see Table V) is small compared with 1.0. On the other hand the excellent agreement between the **first order approximation** and the **complete calculation** over a large range for our three cones may perhaps justify its practical use for bodies which are not cones.

Summary

A method is developed for calculating the air flow and pressure in the neighbourhood of a projectile with a conical head. The calculations are carried out for cones of vertical angle 20°, 40° and 60°. It is shown that the **shock wave is attached** to the point of the projectile provided its velocity is greater than 1.035*a* in the case of the 20° cone, 1.18*a* for the 40° cone and 1.46*a* for the 60° cone; *a* is the speed of sound. At lower speeds the shock wave is **detached** from the point and travels ahead of it. This conclusion is

verified accurately in the case of the 60° cone by means of photographs of bullets in flight.

The calculated pressures on the head of the projectile are compared with those measured in a high speed wind tunnel and good agreement is found.

The complete solution here developed is compared with v. Karman and Moore's approximate theory and good agreement is found in the case of the 20° cone over a limited range of speeds. The approximate theory does not apply to the cones of wider angle.