CHAPTER XII

SOME SELF-SIMILAR PROCESSES IN GAS DYNAMICS

1. Introduction

§ 1. Transformation Groups Admissible by Equations of Gas Dynamics

In Chapter I we already became familiar with several examples of self-similar motions (with the self-similar wave of rarefaction, with the problem about the powerful explosion.[1] In this chapter we will study in detail the self-similar motion of one of two basic types. In the introductory section of the chapter, we will show how in equations of gas dynamics the possibility of existence of self-similar solutions is expressed and we will give the common characteristics, of self-similar motions. It is expedient preliminarily to become familiar with the common group properties of equations of gas dynamics.

We will consider one-dimensional adiabatic motions of an ideal gas with constant heat capacity, i.e., motions possessing plane, cylindrical, or spherical symmetries. Let us write out a system of equations for these types of motions. In continuity equation (1.2) we open the sign of divergence and present the equation in a single form, common for all three forms of symmetry; furthermore, we will divide the entire equation by density ρ. In adiabatic equation (1.13) we place the expression for entropy (1.14) (replacing specific volume by density). Equation of motion (1.6) will remain without changes. Let us obtain the following system of equations for density, pressure, and speed as coordinate and time functions:

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{u} = 0}$$

$$\boxed{\begin{array}{l} \dfrac{DS}{Dt} = 0 \\ S = c_v \ln pV^{\gamma} + const. \end{array}},$$

$$\boxed{\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p}$$

$$\left.\begin{array}{r} \dfrac{\partial \ln \rho}{\partial t} + u \dfrac{\ln \rho}{\partial r} + \dfrac{\partial u}{\partial r} + (\nu - 1)\dfrac{u}{r} = 0 \\ \dfrac{\partial u}{\partial t} + u \dfrac{\partial u}{\partial r} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial r} = 0 \\ \dfrac{\partial}{\partial t} \ln p\rho^{-\gamma} + u \dfrac{\partial}{\partial r} \ln p\rho^{-\gamma} = 0 \end{array}\right\}. \qquad (12.1)$$

Number ν in the continuity equation is equal to ν = 1, 2, 3 for plane, cylindrical, and spherical cases, correspondingly. Variable r plays the role of coordinate x in the plane case and radius in the cylindrical and spherical cases.

Equations (12.1) allow several transformation groups which we will now enumerate. It is assumed that simultaneously with the

[1] In Chapter X we considered self-similar problems of the theory of propagation of heat by the mechanism of thermal conduction in a motionless substance.

transformations in the equations analogous transformations are made both in the Initial and boundary conditions of the problem.

(1) Time t enters the equations only under the sign of the differential, and consequently, the time shift, accomplished by means of introduction of the new variable $t' = t + t_0$, does not change the equations. The possibility of a time shift is connected with the arbitrariness in selection of the beginning of the time reading.

(2) In the plane case ($\nu = 1$) the coordinate also enters the equations only under the sign of the differential. Therefore in the plane case there is also possible a coordinate shift connected with the arbitrariness in selection of the beginning of the coordinate reading. Introduction of variable $x' = x + x_0$ does not change the equations. In the spherical and cylindrical cases this is impossible, since the radius enters the continuity equation not only under the sign of the differential.

Equations of gas dynamics contain five dimensional magnitudes ρ , p, u, r, t, from which **three** possess independent dimension. For instance, if one were to select as the basic dimensional magnitudes density, coordinate, and time, the dimensions of speed and pressure are in the form of [u] = [r]/[t]; [p] – [ρ][r^2]/[t]. In accordance with the existence of three independent dimensional magnitudes the equations permit **three independent transformation groups of similarity**, which are connected with the arbitrariness in selection of units of measurement of the basic dimensional magnitudes.

(1) Let us assume that functions $\rho = f_1(r,t)$, $p = f_2(r,t)$ and $u = f_3(r,t)$ constitute the solution of equations for a certain defined motion. Let us change the scale of density, not changing the scales of coordinate and time, for which we introduce new variables $\rho' = k\rho$, $p' = kp$, leaving the rest without change. The equations will not be changed. If we simultaneously change in the same form the initial and boundary conditions, increasing density and pressure k times, the new motion will be described by the functions

$$\rho' = kf_1(r,t) , \quad p' = kf_2(r,t) , \quad u = f_3(r,t) .$$

The new motion is like the old, differing only by scales of density and pressure.

(2) Let us change the scale of length, not changing the scales of density and time. The equations do not change, if we cross in them to new variables: $r' = mr$, $u' = mu$, $p' = m^2 p$, leaving the others, ρ and t,

without change: $\rho' = \rho$, $t' = t$. This means that if some motion is described by functions $\rho = f_1(r,t)$, $p = f_2(r,t)$, $u = f_3(r,t)$, by means of simple change of scales it is also possible to describe the new motion, in which the distances and speeds are increased *m* times, and pressure is increased m^2 times (density remains constant). The solution for the new motion are the functions:

$$\rho' = f_1(r',t), \quad p' = m^2 f_2(r',t), \quad u' = mf_3(r',t).$$

(3) Finally, we will change the scale of time, not changing the scales of length and density. The equations allow this transformation:

$$t' = nt, \quad u' = \frac{u}{n}, \quad p' = \frac{p}{n^2}, \quad \rho' = \rho, \quad r' = r.$$

This means that if in initial and boundary conditions the speeds are decreased *n* times and pressure n^2 times, leaving density constant, the new process will be like the old one, but will only be *n* times slower.

By means of consecutive application of three transformation groups of similarity one can obtain solutions for an infinite number of new motions with modified scales of density, length, and time. In particular, if we simultaneously extend length and time an identical number of times $r' = lr$, $t' = lt$, the solution will remain constant. Such transformation is equivalent to consecutive application of transformations 2) and 3) with $m = n = l$. In symbolic form this is possible to record so :

$$u(r,t) \to lu(lr,t) \to \frac{1}{l} \cdot lu(lr,lt) = u(lr,lt)$$

and analogously for other functions, ρ and *p*.

§ 2. Self-Similar Motions

In the preceding paragraph it was shown that equations of gas dynamics allow similarity transformations, i.e., there are possible different motions which are similar to each other and can be obtained from one another by means of change of the main scales of length, time and density. Regarding, however, the given motion, it can be described by the most diverse functions of two variables *r* and *t*: $\rho(r,t)$, $p(r, t)$, $u(r, t)$, including also the parameters which enter the initial and boundary conditions of the problem (and adiabatic exponent γ).

There exist, however, such motions, the distinctive property of which is the similarity conserved in the actual motion. Such motions are called ***self-similar***. Distribution of any of gas-dynamic magnitudes with

respect to coordinate, let us say, pressure p, in self-similar motion evolves in time in such a way that are changed only the scale of pressure $P(t)$ and coordinate scale of the region enveloped by the motion $R(t)$, but the shape of the profile of pressure remains constant. By means of extension and reduction of scales P and R it is possible to reach exact coincidence of curves $p(r)$, responding to different moments of time t. Function $p(r, t)$ can be presented in the form of $p(r,t) = P(t)\pi(r/R)$, where the dimensional scales P and R somehow depend on time, and dimensionless ration $p/P = \pi(r/R)$ is a "universal" (in the sense of independence on time) function of the new dimensionless coordinate $\xi = r/R$. By extending and reducing scales P and R in accordance with their dependence on time, it is possible from the "universal" function $\pi(\xi)$ to obtain a true curve of pressure distribution with respect to coordinate $p(r)$ for any moment of time t. Likewise expressed are the other gas dynamic magnitudes: density and speed.

For self-similar motions the system of equations of gas dynamics in partial derivatives reduces to a system of ordinary differential equations with respect to new unknown functions of self-similar variables $\xi = r/R$.

We shall work out these equations. For this we shall present the solution of partial differential equations (12.1) In the form of the products of scale functions by new unknown functions of the new self-similar variable ξ:

$$\xi = \frac{r}{R}, \quad R = R(t). \qquad (12.2)$$

Scales of pressure, density, speed and length are not all independent upon one another. If one were to select as the main scales of length R and density ρ_0, as the scale of speed it is possible to take the magnitude $\frac{dR}{dt} \equiv \dot{R}$, and as the scale of pressure $\rho_0 \dot{R}^2$. This does not limit the generality of the solution and the scale is determined with an accuracy of the numerical coefficient, which is always possible to include in the new unknown function. We shall find the solution in the form

$$p = \rho_0 \dot{R}^2 \pi(\xi), \quad \rho = \rho_0 g(\xi), \quad u = \dot{R} v(\xi), \qquad (12.3)$$

where π, g and v are new, dimensionless functions of self-similar variable ξ, for which one should compose differential equations.

These functions are sometimes called ***representatives*** of pressure, density and speed, correspondingly. Scales of R, ρ_0 and $\dot{R}$ somehow depend on time, although in an unknown manner.

We shall place expression (12.3) in equation (12.1), take into account the determination of self-similar variable (12.2), and shall use the rules of differentiation of the type:

$$\left.\begin{aligned}\frac{\partial\rho}{\partial t}&=\frac{d\rho_0}{dt}\cdot g-\rho_0\frac{dg}{d\xi}\frac{r}{R^2}\frac{dR}{dt}=\dot{\rho}_0 g-\rho_0 g'\xi\frac{\dot{R}}{R}\\ \frac{\partial\rho}{\partial r}&=\frac{\rho_0 g'}{R}\end{aligned}\right\}$$

(differentiation of scales in time is designated by the dot, and differentiation of representatives with respect to self-similar variable, by the prime). As a result, after simple transformations we obtain the equations:

$$\left.\begin{aligned}\frac{\dot{\rho}_0}{\rho_0}+\frac{\dot{R}}{R}\left[v'+(v-\xi)(\ln g)'+(v-1)\frac{v}{\xi}\right]=0\\ \frac{R\ddot{R}}{\dot{R}^2}v+(v-\xi)v'+\frac{\pi'}{g}=0\\ \frac{R}{\dot{R}}\frac{d}{dt}(\ln\rho_0{}^{1-\gamma}\dot{R}^2)+(v-\xi)(\ln\pi g^{-\gamma})'=0\end{aligned}\right\}. \qquad (12.4)$$

So that presentation (12.3) is meaningful and it is possible to write differential equations for new unknown functions $\pi(\xi)$, $g(\xi)$ and $v(\xi)$, it is necessary to separate variables t and ξ in equations (12.4). For this, in the second equation one should put $\frac{R\ddot{R}}{\dot{R}^2}=\text{const}$, whence (when $\text{const}\neq 1$)

$$R=At^{\alpha}. \qquad (12.5)$$

Here A and α are certain constants (A - dimensional, α - number). In the first equation of (12.4) it is necessary to put $\frac{\dot{\rho}_0}{\rho_0}=\text{const}\frac{\dot{\mathrm{R}}}{\mathrm{R}}$, which gives

$$\rho_0=Bt^{\beta}, \qquad (12.6)$$

where B and β are also constants. The first member in the third equation of (12.4) then is turned into a constant automatically. Thus, all scales in self-similar motion depend on time according to root laws, and the self-similar variable has the form[2]

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[2] As noted by K. P. Stanyukovich [i], in addition to root self similarity, exponential self-similarity is also possible, in which $R=A'e^{mt}$, $\rho_0=B'e^{nt}$ and $\xi=re^{-mt}/A'$, where A', B', m and n are constants. The exponential solution satisfies the equation $R\ddot{R}/\dot{R}^2=\text{const}$ for const=1. The majority of practically interesting problems have a

$$\xi = \frac{r}{R} = \frac{r}{At^{\alpha}} . \qquad (12.7)$$

Equations (12.4) are now transformed into a system of three ordinary differential equations with respect to three unknown functions $\pi(\xi)$, $g(\xi)$ and $v(\xi)$. The system contains exponents: constant numbers α and β. In an analogous way the initial and boundary conditions of the problem, will be converted to dimensionless form. They are converted into conditions for functions π, g, and v. Here we shall write out the system of equations in the common form. The equations will be written subsequently in reference to specific problems. In many motions the scale of density ρ_0 is constant (exponent $\beta = 0$). This takes place, for instance, in all cases when a shock wave (or wave of rarefaction) spreads through an initial gas of constant density.

Exponent β usually differs from zero in those problems in which the density of initial gas is distributed in space by root law of the type $\rho_{00} = \text{const} \cdot \text{r}^{\delta}$. In these cases exponent β is determined through known exponent δ and α (when $\delta = 0$, $\beta = 0$). Thus, in the system of equations for function π, g, and v (and in boundary conditions) there enters only one new parameters: the exponent of self-similarity α.

The exponents in scale functions in a simple manner are connected with exponents α and β (i.e., α and δ). For instance, when the scale of density is constant ($\beta = 0$, $\rho_0 = \text{const}$),

$$R \sim t^{\alpha}, \quad \dot{R} \sim t^{\alpha-1}, \quad P = \rho_0 \dot{R}^2 \sim t^{2(\alpha-1)}.$$

Inasmuch as the scale of length R in a simple manner is connected with time, the scales of speed, density and pressure can be considered as functions not of time, but of scale of length R; with help of the relationship $R \sim t^{\alpha}$ we find

$$\dot{R} \sim t^{\alpha-1} \sim R^{(\alpha-1)/\alpha}, \quad \rho_0 \sim t^{\beta} \sim R^{\beta/\alpha},$$

$$P \sim \rho_0 \dot{R}^2 \sim t^{\beta+2(\alpha-1)} \sim R^{[\beta+2(ga-1)]/\alpha}.$$

From expressions for scale of density $\rho_0 \sim t^{\beta} \sim R^{\beta/\alpha}$ and the law of distribution of initial density in space $\rho_{00} = \text{const} \cdot \text{r}^{\delta}$, it is clear that $\rho_0 = \rho_{00}(R)$; for instance, as the scale of density ρ_0 serves the initial density of gas at the point where the shock wave is at the time t (R is the coordinate of the front of the shock wave). From this follows the above-mentioned connection of exponents β and δ: $\beta = \alpha\delta$.

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power-law character.

When $\beta = 0$, and $\rho_0 = \text{const}$, functions p, ρ and u by formulas (12.3) can be written in any of the equivalent forms:

$$\left.\begin{array}{l} p = \text{const} \cdot \mathrm{t}^{2(\alpha\text{-}1)}\pi(\xi) = \text{const} \cdot \mathrm{R}^{2(\alpha\text{-}1)/\alpha}\pi(\xi) \\ u = \text{const} \cdot \mathrm{t}^{\alpha\text{-}1}\nu(\xi) = \text{const} \cdot R^{(\alpha-1)/\alpha}\nu(\xi) \\ \rho = \text{const} \cdot g(\xi) \end{array}\right\}. \qquad (12.8)$$

§ 3. Conditions of Self-Similar Motion

It is natural to pose the question: what requirements must be satisfied by the conditions of a problem so that motion is self-similar? For the answer to this question it follows to draw-on considerations of dimension.

Equations of gas dynamics (12.1) besides variables of functions *p*, ρ, and *u* and independent variables *r* and *t*, do not contain any dimensional parameters (the only parameter, γ, is dimensionless). Dimensional parameters enter the initial and boundary conditions of the problem. This also gives the possibility to construct functions *p*(*r*, *t*) and ρ (*r*, *t*), since all five variables, *p*, ρ, *u*, *r*, and *t* have different dimensions, whereby three of them are independent. Inasmuch as the dimensions of pressure and density contain the unit of mass, at least one of the parameters of the problem also should contain the unit of mass.

In many cases this is the constant initial density of gas ρ_0, which has the dimension of ML^{-3}. In a number of problems the initial density is distributed in space by root law $\rho_{00} = b\mathrm{r}^{\delta}$. In this case it is the parameter *b*, with dimension [*b*] = $ML^{-3-\delta}$. Let us designate the parameter containing the unit of mass through *a*. In the most common case its dimension is [*a*] = $ML^{k}T^{s}$. Considering of the dimensions of functions: [*p*] = $ML^{-1}T^{-2}$, [ρ]=ML^{-3}, [u]= LT^{-1}, it is possible, without any loss of generality, to present then in the form proposed by L. I. Sedov [2]:

$$\left.\begin{array}{l} p = \dfrac{a}{r^{k+1}t^{s+2}}P \\ \rho = \dfrac{a}{r^{k+3}t^{s}}G \\ u = \dfrac{r}{t}V \end{array}\right\}, \qquad (12.9)$$

where *P*, *Q*, and *V* are dimensionless functions of independent variables, which depend on dimensionless combinations containing *r* and *t* and parameters of the problem.

In the general case there are two dimensionless variables: $\frac{r}{r_0}$ and $\frac{t}{t_0}$, where r_0 and t_0 are parameters with dimensions of length and time, either directly enter the conditions of the problem or can be composed by means of combination of parameters of another dimension. Functions *P*, *G*, *V* then depend on *r* and *t* separately and the problem is not self-similar.

It is possible to give a great number of examples of similar motions. Let us refer to one: the problem about a **wave of rarefaction** which appears when a piston is advanced from a gas with variable speed $u_1 = U(1 - e^{-t/\tau})$ (see § 10, Chapter I). In this example the role of parameter *a* is played by the constant initial density of the gas ρ_0. Furthermore the problem contains dimensional parameters [τ] = *T*; [U] = LT^{-1}, and initial speed of sound [c_0] = LT^{-1} (or initial pressure p_0; $c_0{}^2 = \gamma \frac{p_0}{\rho_0}$). Dimensionless variables can be, for instance, $\frac{t}{\tau}$ and $\frac{r}{c_0\tau}$, or $\frac{r}{U\tau}$ ($r_0 = c_0\tau$ or $U\tau$).

If from the parameters of the problem it is Impossible to compose scales of length and time, the variables *r* and *t* cannot enter functions *P*, *G* and *V* separately; the functions can depend only on a dimensionless combination composed from *r* and *t*, $\xi = \frac{r}{At^{\alpha}}$, where *A* is a certain parameter of dimension [A] = $LT^{-\alpha}$. Expressions (12.9) the takes the form

$$\left.\begin{aligned} p &= \frac{a}{r^{k+1} t^{s+2}} P(\xi) \\ \rho &= \frac{a}{r^{k+3} t^{s}} G(\xi) \\ u &= \frac{r}{t} V(\xi) \end{aligned}\right\}. \qquad (12.10)$$

In this case the problem is self-similar and expressions (12.10) are equivalent to expressions (12.3), differing from the latter only by the form of representative functions.

We shall demonstrate this in an example of self-similar motions with constant scale of density. With this, $a = \rho_0$, *k* = -3, *s* = 0, so that expressions (12.10) take on this form:

$$\left.\begin{aligned} p &= \frac{\rho_0}{r^{-2} t^2} P(\xi) = \rho_0 \frac{r^2}{t^2} P(\xi) \\ \rho &= \frac{\rho_0}{r^0 t^0} G(\xi) = \rho_0 G(\xi) \\ u &= \frac{r}{t} V(\xi) \end{aligned}\right\}. \tag{12.11}$$

Putting here $r = \xi R$ and noticing that $\dot{R} = \alpha R / t$, we find that formulas (12.11) and (12,5) are equivalent if

$$\left.\begin{aligned} P(\xi) &= \alpha^2 \frac{\pi(\xi)}{\xi^2} \\ G(\xi) &= g(\xi) \\ V\xi) &= \alpha \frac{v(\xi)}{\xi} \end{aligned}\right\}. \tag{12.12}$$

Study of self-similar motions presents great interest. The possibility of reduction of a system of partial differential equations to a system of ordinary differential equations for new representative functions, extraordinarily simplifies the problem from the mathematical standpoint and in a number of cases permits the finding of exact analytic solutions. Furthermore, frequently self-similar solutions constitute the limits which the solutions of non self-similar problems asymptotically tend to. Subsequently this position will be clarified in the examination of specific problems.

§ 4. Two Types of Self-Similar Solutions

There exist two sharply different types of self-similar solutions. Solutions of the **first type** possess the property that the index of self-similarity α, and together with it, the exponents at t or R in all scales, are determined from considerations of **dimensions** or from **laws of conservation**. The exponents are then fractions with integral numerators and denominators. In problems of this type there always are **two parameters** with independent dimension. From these parameters there is composed a parameter whose dimension contains the unit of mass and (see formula (12.10)), and another parameter A, which contains only units of length and time. With the help of the second parameter A it is also possible to construct a dimensionless combination, i.e., self-similar variable $\xi = r / At^{\alpha}$. The dimension of parameter A, $LT^{-\alpha}$, is determined by the index of self-similarity α. Two motions of such type were considered in Chapter I: the problem about the self-similar **wave of rarefaction** (§ 11) and the problem about the **strong explosion** (§25). In the first case the two independent

dimensional parameters are initial density ρ_0 and pressure of gas p_0. From them it is possible to compose a dimensional parameter not containing the unit of mass; the initial speed of sound $c_0 = \left(\frac{\gamma p_0}{\rho_0}\right)^{1/2}$.

The role of parameter A is played by the speed of sound c_0. Correspondingly,

$$\xi = \frac{r}{c_0 t}, \quad \alpha = 1 .$$

In the problem concerning the strong explosion, the parameters are initial gas density $\rho_0 \sim ML^{-3}$ and energy of explosion $E \sim ML^2T^{-2}$. The energy is always equal to total energy of gas enveloped by motion, and as a result an energy integral appears in the problem. (We recall that in the problem about the strong explosion the initial pressure p_0 and speed of sound c_0 are assumed to be equal to zero, i.e., these magnitudes are not parameters of the problem). From parameters ρ_0 and E there is composed a parameter not containing mass, $A = \left(\frac{E}{\rho_0}\right)^{1/5} \sim LT^{-2/5}$, so that the self-similar variable is $\xi = \frac{r}{(E/\rho_0)^{1/5} t^{2/5}}$; $\alpha = \frac{2}{5}$.

In an intense explosion in a medium with variable initial density $\rho_{00} = br^{\delta}$, the parameters are the energy of the explosion $E \sim ML^2T^{-2}$ and coefficient $b \sim ML^{-3-\delta}$. From them it is possible to compose parameter A, not containing mass

$$A = \left(\frac{E}{b}\right)^{1/(5+\delta)} \sim LT^{-2/(5+\delta)} .$$

The self-similar variable has the form

$$\xi = \frac{r}{\left(\frac{E}{b}\right)^{1/(5+\delta)} t^{2/(5+\delta)}}; \quad \alpha = \frac{2}{2+\delta} .$$

(A self-similar problem about an explosion in a medium with variable density was considered by L. I. Sedov [2]). A self-similar problem of the same type is the one about propagation of a thermal wave from the place where a definite amount of energy was released (see Chapter X).

As was shown in § 2, the index of self-similarity enters the system of differential equations for representatives as a parameter. Inasmuch as in self-similar problems of the considered type the number α is immediately found from considerations of dimension (or laws of

conservation), the matter reduces to integration of the system of equations with known boundary conditions.

In self-similar problems of the **second type**, exponent α is impossible to find from considerations of dimension or laws of conservation without solution of equations. In this case the actual determination of the index of self-similarity requires integration of ordinary differential equations for representative functions. It turns out that the index is found from the condition that the integral curve passes through a singular point, without which it is not possible to satisfy the boundary conditions.

Examples of self-similar motions of the second type are problems of an imploding shock wave and of an impulsive load, which will be discussed below.

Consideration of solutions of specific problems, belonging to the second type, shows that in all these cases in initial conditions of the problem there is only one dimensional parameter containing the unit of mass, and there is no second one, with help of which it would have been possible to form parameter A. This also deprives us of the possibility to establish number α with respect to dimension of parameter A. In fact, the problem of course is peculiar to a certain dimensional parameter $A \sim LT^{-\alpha}$; otherwise it would have been impossible to compose the dimensionless combination $\xi = r / At^{\alpha}$. However, the dimension of this parameter (i.e., number α) is not dictated by the initial conditions of the problem, but is found from solution of the equation. The numerical value of parameter A is impossible to find from equations of self-similar motion. It can be determined only by knowing how the given motion appeared. Thus, for instance, if the self-similar motion appeared as a result of some non self-similar flow, which asymptotically went into a self-similar regime, the magnitude A can be found only be means of numerical solution of the full, non self-similar problem, when the process of transition of non self-similar motion to self-similar has been traced. In greater detail these positions will be explained in the examination of specific problems.

Self-similar motions of the first type, in which the index of self-similarity is determined from considerations of dimensions, in detail were investigated by L. I. Sedov. Inasmuch as there is already the book of L. I. Sedov [2], in which he gives an exhausting description of

these motions and the solution of a number of specific problems, in this chapter we will not remain on self-similar motions of the first type and will be occupied with the study of motions of only the **second type**.

2. Implosion of a spherical shock wave and the collapse of bubbles in a liquid

§ 5. Statement of the problem of an imploding shock wave

(continued)