述語論理 証明問題　自然演繹を用いて次の連式を証明せよ.

問題 1

[1] $Fa, \forall x(Fx \to Gx) \vdash Ga$
[2] $Ga, \forall x(Fx \to {\sim}Gx) \vdash {\sim}Fa$
[3] ${\sim}Fb, \forall x(Gx \to Fx) \vdash {\sim}Gb$
[4] $\forall x((Fx \,\&\, Gx) \to Hx), {\sim}Hb, Fb \vdash {\sim}Gb$
[5] $\forall x(Fx \to ({\sim}Gx \lor Hx)), Fa, Ga \vdash Ha$
[6] $\forall x((Fx \,\&\, {\sim}Gx) \to Hx), Fa, {\sim}Ha \vdash Ga$
[7] $\forall x(Fx \to Gx), \forall x(Hx \to Fx) \vdash Ga \lor {\sim}Ha$
[8] $\forall x(Fx \to Hx), \forall x(Hx \to {\sim}Gx) \vdash \forall xFx \to {\sim}\forall xGx$

問題 2

[1] $\forall x(Fx \to Gx), \forall x{\sim}Gx \vdash \forall x{\sim}Fx$
[2] $\forall x((Fx \lor Gx) \to Hx), \forall x{\sim}Hx \vdash \forall x{\sim}Fx$
[3] $\forall x(Fx \to {\sim}Gx), \forall xFx, \forall x(Hx \lor Gx) \vdash \forall x(Fx \,\&\, Hx)$
[4] $\forall x(Fx \to Hx), \forall x({\sim}Gx \to {\sim}Hx) \vdash \forall xFx \to \forall xGx$
[5] $\forall x(Fx \to {\sim}Hx), \forall x(Gx \lor Hx) \vdash \forall x(Fx \to Gx)$
[6] $\forall x(Fx \to Gx), \forall x(Gx \to {\sim}Hx) \vdash \forall x(Fx \to {\sim}Hx)$
[7] $\forall x((Fx \,\&\, Gx) \to Hx) \vdash \forall xFx \to \forall x(Gx \to Hx)$
[8] $\forall x(Fx \to Gx), \forall x({\sim}Hx \to Fx) \vdash \forall x(Gx \lor Hx)$
[9] $\forall x(Fx \,\&\, Gx) \dashv\vdash \forall xFx \,\&\, \forall xGx$
[10] $\forall x(Fx \to Hx), \forall x(Gx \to Ix) \vdash \forall x((Fx \lor Gx) \to (Hx \lor Ix))$
[11] $\forall xFx \lor \forall xGx \vdash \forall x(Fx \lor Gx)$
[12] $\forall x(Fx \to (Gx \,\&\, Hx)) \dashv\vdash \forall x(Fx \to Gx) \,\&\, \forall x(Fx \to Hx)$
[13] $\forall x((Fx \lor Gx) \to Hx) \dashv\vdash \forall x(Fx \to Hx) \,\&\, \forall x(Gx \to Hx)$
[14] $\forall x(Fx \to \forall xGx) \vdash \forall x(Fx \to \forall x(Gx \lor Hx))$

問題 3

[1] $\forall xFx \vdash \exists xFx$
[2] $\forall x(Fx \to Gx), \forall x{\sim}Gx \vdash \exists x{\sim}Fx$
[3] $\forall x(Fx \to Gx), \exists x{\sim}Gx \vdash \exists x{\sim}Fx$
[4] $\forall xFx \,\&\, \exists xGx \vdash \exists x(Fx \,\&\, Gx)$
[5] $\forall x(Fx \to (Gx \,\&\, Hx)), \exists xFx \vdash \exists xHx$
[6] $\forall x((Fx \lor Gx) \to Hx), \exists x{\sim}Hx \vdash \exists x{\sim}Fx$
[7] $\forall x(Gx \to {\sim}Hx), \exists x(Fx \,\&\, Gx) \vdash \exists x(Fx \,\&\, {\sim}Hx)$
[8] $\forall x(Hx \to Gx), \exists x(Fx \,\&\, {\sim}Gx) \vdash \exists x(Fx \,\&\, {\sim}Hx)$
[9] $\forall x(Gx \lor Hx), \exists x(Fx \,\&\, {\sim}Gx) \vdash \exists x(Fx \,\&\, Hx)$
[10] $\forall x(Gx \to Hx) \vdash \exists x(Gx \,\&\, Fx) \to \exists x(Fx \,\&\, Hx)$
[11] $\exists x(Fx \lor Gx), \forall x(Gx \to Hx) \vdash \exists x(Fx \lor Hx)$
[12] $\exists xFx, {\sim}\exists x(Fx \,\&\, Gx) \vdash \exists x{\sim}Gx$
[13] $\exists x(Fx \,\&\, Gx) \vdash \exists xFx \,\&\, \exists xGx$
[14] $\exists x(Fx \lor Gx) \dashv\vdash \exists xFx \lor \exists x\, Gx$

問題 4

[1] ${\sim}\forall xFx \dashv\vdash \exists x{\sim}Fx$
[2] ${\sim}\exists xFx \dashv\vdash \forall x{\sim}Fx$
[3] $\forall x(Fx \to {\sim}Gx) \dashv\vdash {\sim}\exists x(Fx \,\&\, Gx)$
[4] $\forall x(Fx \leftrightarrow Gx) \dashv\vdash \forall x(Fx \to Gx) \,\&\, \forall x(Gx \to Fx)$
[5] $\forall x(Fx \leftrightarrow Gx) \vdash \forall xFx \leftrightarrow \forall xGx$
[6] $\forall x(Fx \leftrightarrow Gx) \vdash \exists xFx \leftrightarrow \exists xGx$
[7] $\forall x(P \to Fx) \dashv\vdash P \to \forall xFx$
[8] $\exists x(P \to Fx) \dashv\vdash P \to \exists xFx$
[9] $\forall x(P \,\&\, Fx) \dashv\vdash P \,\&\, \forall xFx$

[10] $\exists x(P \,\&\, Fx) \dashv\vdash P \,\&\, \exists xFx$
[11] $\forall x(P \vee Fx) \dashv\vdash P \vee \forall xFx$
[12] $\exists x(P \vee Fx) \dashv\vdash P \vee \exists xFx$
[13] $\forall x(Fx \to P) \dashv\vdash \exists xFx \to P$
[14] $\exists x(Fx \to P) \dashv\vdash \forall xFx \to P$

問題 5
[1] $\exists xFx \to \exists xGx \vdash \exists x(Fx \to Gx)$
[2] $\forall xFx \to \forall xGx \vdash \exists x(Fx \to Gx)$
[3] $\exists xFx \to \forall xGx, \forall x(Gx \vee Hx) \to \forall xJx \vdash \forall x(Fx \to Jx)$
[4] $\forall xFx \to \forall \mathrm{x}Gx \vdash \exists x(Fx \to Gx)$
[5] $\exists xFx \to \exists \mathrm{x}Gx \vdash \exists x(Fx \to Gx)$
[6] $\forall x((Gx \vee Hx) \to Fx), \exists xGx, \forall x(Fx \to \forall xHx) \vdash \forall xFx$
[7] $\forall x(Fx \to Gx) \vee \exists x(Fx \,\&\, Hx), \forall x(Kx \to ({\sim}Jx \vee {\sim}Hx)), \forall x(Fx \to (Kx \,\&\, Jx)), \exists xFx \vdash \exists x(Gx \,\&\, Fx)$
[8] $\exists x(Fx \,\&\, {\sim}Gx), \forall x(Fx \to Hx), \forall x((Jx \,\&\, Kx) \to Fx), \exists x(Hx \,\&\, {\sim}Gx) \to \forall x(Kx \to {\sim}Hx) \vdash \forall x(Jx \to {\sim}Kx)$
[9] $\forall x(Fx \to (Gx \vee Hx)), \forall x((Gx \vee Hx) \to Kx), {\sim}\exists x(Kx \,\&\, Gx), {\sim}\exists Fx \to \exists xGx \vdash \exists x(Fx \,\&\, Hx)$
[10] $\forall x((Fx \,\&\, Hx) \to Gx) \vdash \forall x((Fx \,\&\, Gx) \to {\sim}Hx) \to {\sim}\exists x(Fx \,\&\, Hx)$