

命題論理 証明問題 自然演繹を用いて次の連式を証明せよ.

問題 1

- [1] $P \rightarrow Q, Q \rightarrow R, P \vdash R$
- [2] $P, P \rightarrow Q, P \rightarrow (Q \rightarrow R) \vdash R$
- [3] $P \rightarrow (P \rightarrow Q), P \vdash Q$
- [4] $(P \& \sim Q) \rightarrow (R \rightarrow S), P \& \sim Q, (R \rightarrow S) \rightarrow U \vdash U$
- [5] $P \rightarrow \sim Q, R \rightarrow Q, P \vdash \sim R$
- [6] $R \rightarrow Q, \sim Q, P \rightarrow R \vdash \sim P$
- [7] $Q \rightarrow (P \rightarrow R), \sim R, Q \vdash \sim P$
- [8] $P, P \rightarrow (Q \rightarrow R), P \rightarrow \sim R \vdash \sim Q$
- [9] $\sim(R \rightarrow S), (P \vee Q) \rightarrow (R \rightarrow S), \sim R \rightarrow (P \vee Q) \vdash \sim \sim R$

問題 2

- [1] $P \rightarrow \sim \sim Q, P \vdash Q$
- [2] $\sim Q, \sim R \rightarrow Q, R \rightarrow P \vdash P$
- [3] $P \rightarrow \sim Q, Q \vdash \sim P$
- [4] $\sim \sim Q \rightarrow P, \sim P \vdash \sim Q$
- [5] $\sim P \rightarrow \sim Q, Q \vdash P$
- [6] $P, P \rightarrow Q, \sim(Q \rightarrow R) \rightarrow \sim P \vdash R$

問題 3

- [1] $P \rightarrow Q \vdash \sim Q \rightarrow \sim P$
- [2] $P \rightarrow \sim Q \vdash Q \rightarrow \sim P$
- [3] $P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$
- [4] $P \vdash (P \rightarrow Q) \rightarrow Q$
- [5] $P \rightarrow Q \vdash (Q \rightarrow R) \rightarrow (P \rightarrow R)$
- [6] $P \rightarrow (\sim Q \rightarrow R) \vdash \sim R \rightarrow (P \rightarrow Q)$
- [7] $\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
- [8] $P \rightarrow (Q \rightarrow (R \rightarrow S)) \vdash R \rightarrow (P \rightarrow (Q \rightarrow S))$
- [9] $P \vdash (\sim(Q \rightarrow R) \rightarrow \sim P) \rightarrow (\sim R \rightarrow \sim Q)$
- [10] $(P \rightarrow Q) \rightarrow R, P \rightarrow S \vdash (S \rightarrow Q) \rightarrow R$
- [11] $\vdash P \rightarrow P$
- [12] $P \vdash Q \rightarrow P$

問題 4

- [1] $P \& Q \vdash Q \& P$
- [2] $P \& Q, P \rightarrow (Q \rightarrow R) \vdash R$
- [3] $P \vdash Q \rightarrow (P \& Q)$
- [4] $(P \& Q) \rightarrow R \vdash P \rightarrow (Q \rightarrow R)$
- [5] $P \rightarrow (Q \rightarrow R) \vdash (P \& Q) \rightarrow R$
- [6] $P \& Q \vdash P \vee R$
- [7] $P \& Q, (S \vee Q) \rightarrow R \vdash R$
- [8] $P \& (Q \& R) \vdash (P \& Q) \& R$
- [9] $(P \rightarrow Q) \& (P \rightarrow R) \vdash P \rightarrow (Q \& R)$
- [10] $P \rightarrow Q, R \rightarrow S \vdash (P \& R) \rightarrow (Q \& S)$
- [11] $P \vdash P \& P$

問題 5

- [1] $P \rightarrow (Q \vee R), P \& \sim Q \vdash R$
- [2] $P \vee Q, \sim P \& (Q \rightarrow R) \vdash R$
- [3] $(P \vee Q) \vee R, \sim P \rightarrow \sim R \vdash \sim P \rightarrow Q$
- [4] $\sim P \& Q, (Q \vee R) \rightarrow (P \vee S) \vdash \sim P \& S$
- [5] $P \rightarrow (Q \vee R), R \rightarrow \sim P \vdash P \rightarrow Q$
- [6] $P \vee (Q \rightarrow S), \sim S \vee R \vdash (\sim P \& Q) \rightarrow R$
- [7] $Q \vee R, P \vee \sim(Q \rightarrow R) \vdash P$

問題 6

- [1] $Q \vee R, Q \rightarrow P, \sim P \rightarrow \sim R \vdash P$
- [2] $(P \rightarrow R) \& (Q \rightarrow R) \vdash (P \vee Q) \rightarrow R$
- [3] $P \vee Q \vdash Q \vee P$

- [4] $P \vee Q, P \rightarrow R, Q \rightarrow S \vdash R \vee S$

- [5] $P \vee (Q \vee R) \vdash (P \vee Q) \vee R$
- [6] $P \& (Q \vee R) \vdash (P \& Q) \vee (P \& R)$
- [7] $P \vee (Q \& R) \vdash (P \vee Q) \& (P \vee R)$
- [8] $(P \vee Q) \rightarrow R \vdash (P \rightarrow R) \& (Q \rightarrow R)$
- [9] $P \vdash P \vee P$

問題 7

- [1] $P \rightarrow Q, P \rightarrow \sim Q \vdash \sim P$
- [2] $P \rightarrow R, Q \rightarrow \sim R \vdash \sim(P \& Q)$
- [3] $Q \rightarrow \sim P, R \rightarrow P, R \vee \sim Q \vdash \sim Q$
- [4] $P \vee R, \sim Q \rightarrow (\sim P \& \sim R) \vdash Q$
- [5] $\vdash P \vee \sim P$
- [6] $\sim P \rightarrow P \vdash P$
- [7] $P \vee (R \rightarrow Q), \sim(R \& \sim Q) \rightarrow P \vdash P$
- [8] $P \rightarrow R, Q \rightarrow \sim R, P \vdash \sim(P \rightarrow Q)$
- [9] $P \rightarrow \sim R, Q \rightarrow R \vdash P \rightarrow \sim(Q \vee R)$
- [10] $P \& Q, P \rightarrow (Q \rightarrow \sim R) \vdash \sim(Q \rightarrow R)$
- [11] $P \rightarrow R, \sim Q \rightarrow R \vdash (Q \rightarrow P) \rightarrow R$
- [12] $\sim(P \& Q), P \vdash \sim Q$
- [13] $P \rightarrow \sim(Q \& R), P \& R \vdash \sim Q$
- [14] $\sim P \rightarrow (R \& S), \sim(P \vee Q) \vdash R$
- [15] $\vdash \sim(P \& \sim P)$
- [16] $P \& \sim P \vdash Q$

問題 8

- [1] $Q, P \leftrightarrow Q \vdash P$
- [2] $P \rightarrow Q, Q \rightarrow P \vdash P \leftrightarrow Q$
- [3] $P \leftrightarrow Q \vdash \sim P \leftrightarrow \sim Q$
- [4] $(P \vee Q) \leftrightarrow P \vdash Q \rightarrow P$
- [5] $P \leftrightarrow \sim Q, Q \leftrightarrow \sim R \vdash P \leftrightarrow R$

問題 9

- [1] $\sim(P \& Q) \vdash \sim P \vee \sim Q$
- [2] $\sim(P \vee Q) \vdash \sim P \& \sim Q$
- [3] $P \rightarrow Q \vdash \sim P \vee Q$
- [4] $\sim P \rightarrow Q \vdash P \vee Q$
- [5] $\sim(P \rightarrow Q) \vdash P \& \sim Q$

問題 10

- [1] $\sim(P \vee Q), \sim P \rightarrow (R \rightarrow S), \sim R \rightarrow Q \vdash S$
- [2] $P \rightarrow R, Q \rightarrow \sim(R \& S) \vdash (P \& Q) \rightarrow \sim S$
- [3] $S \vee (Q \rightarrow P), \sim P \rightarrow (Q \vee S) \vdash P \vee S$
- [4] $(P \vee Q) \rightarrow V, S \rightarrow P, Q \vee \sim U, S \vee U \vdash V$
- [5] $\sim(P \vee Q) \rightarrow (R \rightarrow S), Q \rightarrow P \vdash R \rightarrow (P \vee S)$
- [6] $(P \& \sim Q) \rightarrow S, \sim P \rightarrow (Q \vee \sim R) \vdash \sim(Q \vee S) \rightarrow \sim R$
- [7] $\sim P \rightarrow \sim R, Q \vee S \vdash \sim(P \& Q) \rightarrow (R \rightarrow S)$
- [8] $\sim P \rightarrow (Q \rightarrow (R \vee S)), \sim(Q \& \sim S) \rightarrow (P \vee R),$
 $\sim Q \rightarrow \sim P \vdash (P \& Q) \vee R$
- [9] $\sim(P \& \sim Q) \vee (\sim R \rightarrow S), \sim S \& \sim Q,$
 $U \rightarrow (\sim S \rightarrow (\sim R \vee P)) \vdash (P \leftrightarrow \sim R) \rightarrow \sim U$
- [10] $P \leftrightarrow Q \vdash (P \& Q) \vee (\sim P \& \sim Q)$