

## 問題 3 解答

[1]  $\forall xFx \vdash \exists xFx$ 

1	(1) $\forall xFx$	A
1	(2) $Fa$	1 $\forall E$
1	(3) $\exists xFx$	2 $\exists I$

[2]  $\forall x(Fx \rightarrow Gx), \forall x\sim Gx \vdash \exists x\sim Fx$ 

1	(1) $\forall x(Fx \rightarrow Gx)$	A
2	(2) $\forall x\sim Gx$	A
1	(3) $Fa \rightarrow Ga$	1 $\forall E$
2	(4) $\sim Ga$	1 $\forall E$
1,2	(5) $\sim Fa$	3,4 MT
1,2	(6) $\exists x\sim Fx$	5 $\exists I$

[3]  $\forall x(Fx \rightarrow Gx), \exists x\sim Gx \vdash \exists x\sim Fx$ 

1	(1) $\forall x(Fx \rightarrow Gx)$	A
2	(2) $\exists x\sim Gx$	A
3	(3) $\sim Ga$	A
1	(4) $Fa \rightarrow Ga$	1 $\forall E$
1,3	(5) $\sim Fa$	3,4 MT
1,3	(6) $\exists x\sim Fx$	5 $\exists I$
1,2	(7) $\exists x\sim Fx$	2,3-6 $\exists E$

[4]  $\forall xFx \& \exists xGx \vdash \exists x(Fx \& Gx)$ 

1	(1) $\forall xFx \& \exists xGx$	A
1	(2) $\forall xFx$	1 $\&E$
1	(3) $\exists xGx$	1 $\&E$
4	(4) $Ga$	A
1	(5) $Fa$	2 $\forall E$
1,4	(6) $Fa \& Ga$	4,5 $\&I$
1,4	(7) $\exists x(Fx \& Gx)$	6 $\exists I$
1	(8) $\exists x(Fx \& Gx)$	3,4-7 $\exists E$

[5]  $\forall x(Fx \rightarrow (Gx \& Hx)), \exists xFx \vdash \exists xHx$ 

1	(1) $\forall x(Fx \rightarrow (Gx \& Hx))$	A
2	(2) $\exists xFx$	A
3	(3) $Fa$	A
1	(4) $Fa \rightarrow (Ga \& Ha)$	1 $\forall E$
1,3	(5) $Ga \& Ha$	3,4 MP
1,3	(6) $Ha$	5 $\&E$
1,3	(7) $\exists xHx$	6 $\exists I$
1,2	(8) $\exists xHx$	2,3-7 $\exists E$

[6]  $\forall x((Fx \vee Gx) \rightarrow Hx), \exists x\sim Hx \vdash \exists x\sim Fx$ 

1	(1) $\forall x((Fx \vee Gx) \rightarrow Hx)$	A
2	(2) $\exists x\sim Hx$	A
3	(3) $\sim Ha$	A
1	(4) $(Fa \vee Ga) \rightarrow Ha$	1 $\forall E$
1,3	(5) $\sim(Fa \vee Ga)$	3,4 MT
1,3	(6) $\sim Fa \& \sim Ga$	5 De M.
1,3	(7) $\sim Fa$	6 $\&E$

1,3	(8) $\exists x \sim Fx$	7 $\exists$ I
1,2	(9) $\exists x \sim Fx$	2,3-8 $\exists$ E

[7]  $\forall x(Gx \rightarrow \sim Hx), \exists x(Fx \& Gx) \vdash \exists x(Fx \& \sim Hx)$

1	(1) $\forall x(Gx \rightarrow \sim Hx)$	A
2	(2) $\exists x(Fx \& Gx)$	A
3	(3) $Fa \& Ga$	A
1	(4) $Ga \rightarrow \sim Ha$	1 $\forall$ E
3	(5) $Fa$	3 &E
3	(6) $Ga$	3 &E
1,3	(7) $\sim Ha$	4,6 MP
1,3	(8) $Fa \& \sim Ha$	5,7 &I
1,3	(9) $\exists x(Fx \& \sim Hx)$	8 $\exists$ I
1,2	(10) $\exists x(Fx \& \sim Hx)$	2,3-9 $\exists$ E

[8]  $\forall x(Hx \rightarrow Gx), \exists x(Fx \& \sim Gx) \vdash \exists x(Fx \& \sim Hx)$

1	(1) $\forall x(Hx \rightarrow Gx)$	A
2	(2) $\exists x(Fx \& \sim Gx)$	A
3	(3) $Fa \& \sim Ga$	A
1	(4) $Ha \rightarrow Ga$	1 $\forall$ E
3	(5) $Fa$	3 &E
3	(6) $\sim Ga$	3 &E
1,3	(7) $\sim Ha$	4,6 MT
1,3	(8) $Fa \& \sim Ha$	5,7 &I
1,3	(9) $\exists x(Fx \& \sim Hx)$	8 $\exists$ I
1,2	(10) $\exists x(Fx \& \sim Hx)$	2,3-9 $\exists$ E

[9]  $\forall x(Gx \vee Hx), \exists x(Fx \& \sim Gx) \vdash \exists x(Fx \& Hx)$

1	(1) $\forall x(Gx \vee Hx)$	A
2	(2) $\exists x(Fx \& \sim Gx)$	A
3	(3) $Fa \& \sim Ga$	A
3	(4) $Fa$	3 &E
3	(5) $\sim Ga$	3 &E
1	(6) $Ga \vee Ha$	1 $\forall$ E
1,3	(7) $Ha$	5,6 MTP
1,3	(8) $Fa \& Ha$	4,7 &I
1,3	(9) $\exists x(Fx \& Hx)$	8 $\exists$ I
1,2	(10) $\exists x(Fx \& Hx)$	2,3-9 $\exists$ E

[10]  $\forall x(Gx \rightarrow Hx) \vdash \exists x(Gx \& Fx) \rightarrow \exists x(Fx \& Hx)$

1	(1) $\forall x(Gx \rightarrow Hx)$	A
2	(2) $\exists x(Gx \& Fx)$	A
3	(3) $Ga \& Fa$	A
1	(4) $Ga \rightarrow Ha$	1 $\forall$ E
3	(5) $Fa$	3 &E
3	(6) $Ga$	3 &E
1,3	(7) $Ha$	4,6 MP
1,3	(8) $Fa \& Ha$	5,7 &I
1,3	(9) $\exists x(Fx \& Hx)$	8 $\exists$ I
1,2	(10) $\exists x(Fx \& \sim Hx)$	2,3-9 $\exists$ E
1	(11) $\exists x(Gx \& Fx) \rightarrow \exists x(Fx \& Hx)$	2-10 CP

[11]  $\exists x(Fx \vee Gx), \forall x(Gx \rightarrow Hx) \vdash \exists x(Fx \vee Hx)$

1	(1) $\exists x(Fx \vee Gx)$	A
2	(2) $\forall x(Gx \rightarrow Hx)$	A
3	(3) $Fa \vee Ga$	A
4	(4) $\sim(Fa \vee Ha)$	A
4	(5) $\sim Fa \ \& \ \sim Ha$	3 De M.
4	(6) $\sim Fa$	3 &E
4	(7) $\sim Ha$	3 &E
2	(8) $Ga \rightarrow Ha$	2 $\forall E$
2,4	(9) $\sim Ga$	7,8 MT
2,3,4	(10) $Fa$	3,9 MTP
2,3,4	(11) $Fa \ \& \ \sim Fa$	6,10 &I
2,3	(12) $\sim\sim(Fa \vee Ha)$	4,11 RAA
2,3	(13) $Fa \vee Ha$	12 DNE
2,3	(14) $\exists x(Fx \vee Hx)$	13 $\exists I$
1,2	(15) $\exists x(Fx \vee Hx)$	1,3-14 $\exists E$

[12]  $\exists xFx, \sim\exists x(Fx \ \& \ Gx) \vdash \exists x\sim Gx$

1	(1) $\exists xFx$	A
2	(2) $\sim\exists x(Fx \ \& \ Gx)$	A
3	(3) $Fa$	A
4	(4) $Ga$	A
3,4	(5) $Fa \ \& \ Ga$	3,4 &I
3,4	(6) $\exists x(Fx \ \& \ Gx)$	5 $\exists I$
2,3,4	(7) $\exists x(Fx \ \& \ Gx) \ \& \ \sim\exists x(Fx \ \& \ Gx)$	2,6 &I
2,3	(8) $\sim Ga$	4,7 RAA
2,3	(9) $\exists x\sim Gx$	8 $\exists I$
1,2	(10) $\exists x\sim Gx$	1,3-9 $\exists E$

[13]  $\exists x(Fx \ \& \ Gx) \vdash \exists xFx \ \& \ \exists xGx$

1	(1) $\exists x(Fx \ \& \ Gx)$	A
2	(2) $Fa \ \& \ Ga$	A
2	(3) $Fa$	2 &E
2	(4) $\exists xFx$	3 $\exists I$
2	(5) $Ga$	2 &E
2	(6) $\exists xGx$	5 $\exists I$
2	(7) $\exists xFx \ \& \ \exists xGx$	4,6 &I
1	(8) $\exists xFx \ \& \ \exists xGx$	1,2-7 $\exists E$

[14]  $\exists x(Fx \vee Gx) \dashv\vdash \exists xFx \vee \exists xGx$

(i)  $\exists x(Fx \vee Gx) \vdash \exists xFx \vee \exists xGx$

1	(1) $\exists x(Fx \vee Gx)$	A
2	(2) $Fa \vee Ga$	A
3	(3) $Fa$	A
3	(4) $\exists xFx$	3 $\exists I$
3	(5) $\exists xFx \vee \exists xGx$	4 $\vee I$
	(6) $Fa \rightarrow (\exists xFx \vee \exists xGx)$	3-5 CP
7	(7) $Ga$	A
7	(8) $\exists xGx$	7 $\exists I$
7	(9) $\exists xFx \vee \exists xGx$	7 $\vee I$
	(10) $Ga \rightarrow (\exists xFx \vee \exists xGx)$	7-9 CP
2	(11) $\exists xFx \vee \exists xGx$	2,6,10 SCD
1	(12) $\exists xFx \vee \exists xGx$	1,2-11 $\exists E$
	(ii) $\exists xFx \vee \exists xGx \vdash \exists x(Fx \vee Gx)$	

1	(1) $\exists xFx \vee \exists x Gx$	A
2	(2) $\exists xFx$	A
3	(3) $Fa$	A
3	(4) $Fa \vee Ga$	3 $\vee I$
3	(5) $\exists x(Fx \vee Gx)$	4 $\exists I$
2	(6) $\exists x(Fx \vee Gx)$	2,3-5 $\exists E$
	(7) $\exists xFx \rightarrow \exists x(Fx \vee Gx)$	2-6 CP
8	(8) $\exists xGx$	A
9	(9) $Ga$	A
9	(10) $Fa \vee Ga$	9 $\vee I$
9	(11) $\exists x(Fx \vee Gx)$	10 $\exists I$
8	(12) $\exists x(Fx \vee Gx)$	8,9-12 $\exists E$
	(13) $\exists xFx \rightarrow \exists x(Fx \vee Gx)$	8-12 CP
1	(14) $\exists x(Fx \vee Gx)$	1,7,13 SCD