# 真理値分析(1)：真理表

## 4. 恒真式

例

(1)

| $P$ | $\sim P$ | $P \vee \sim P$ |
|---|---|---|
| ⊤ | ⊥ | ⊤ |
| ⊥ | ⊤ | ⊤ |

(2)

| $P$ | $\sim P$ | $P \& \sim P$ | $\sim(P \& \sim P)$ |
|---|---|---|---|
| ⊤ | ⊥ | ⊥ | ⊤ |
| ⊥ | ⊤ | ⊥ | ⊤ |

(3)

| $P$ | $P \to P$ |
|---|---|
| ⊤ | ⊤ |
| ⊥ | ⊤ |

(4)

| $P$ | $Q$ | $Q \to P$ | $P \to (Q \to P)$ |
|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊥ | ⊤ |
| ⊥ | ⊥ | ⊤ | ⊤ |

(5)

| $P$ | $Q$ | $\sim P$ | $\sim P \to Q$ | $P \to (\sim P \to Q)$ |
|---|---|---|---|---|
| ⊤ | ⊤ | ⊥ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊥ | ⊥ | ⊤ | ⊥ | ⊤ |

(6)

| $P$ | $Q$ | $P \to Q$ | $Q \to P$ | $(P \to Q) \vee (Q \to \mathrm{P})$ |
|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊤ | ⊥ | ⊤ |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊤ |

(7)

| $P$ | $Q$ | $P \to Q$ | $(P \to Q) \to P$ | $((P \to Q) \to P) \to P$ |
|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊤ | ⊥ | ⊤ |
| ⊥ | ⊥ | ⊤ | ⊥ | ⊤ |

(8)

| $P$ | $Q$ | $R$ | $Q \to R$ | $P \to (Q \to R)$ | $P \to Q$ | $P \to R$ | $(P \to Q) \to (P \to R)$ | $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ |
|---|---|---|---|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊤ | ⊤ | ⊥ | ⊥ | ⊥ | ⊤ | ⊥ | ⊥ | ⊤ |
| ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |

(9)

| $P$ | $Q$ | $R$ | $P \to Q$ | $Q \to R$ | $(P \to Q) \& (Q \to R)$ | $P \to R$ | $((P \to Q) \& (Q \to R)) \to (P \to R)$ |
|---|---|---|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊤ | ⊤ | ⊥ | ⊤ | ⊥ | ⊥ | ⊥ | ⊤ |
| ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊥ | ⊥ | ⊤ | ⊥ | ⊥ | ⊤ |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |
| ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ |

(10) $((P \& Q) \to R) \leftrightarrow ((Q \to R) \vee {\sim}P)$

| $P$ | $Q$ | $R$ | $P \& Q$ | $(P \& Q) \to R$ | $Q \to R$ | ${\sim}P$ | $(Q \to R) \vee {\sim}P$ | $((P \& Q) \to R) \leftrightarrow ((Q \to R) \vee {\sim}P)$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | ⊥ | T | T |
| T | T | ⊥ | T | ⊥ | ⊥ | ⊥ | ⊥ | T |
| T | ⊥ | T | ⊥ | T | T | ⊥ | T | T |
| T | ⊥ | ⊥ | ⊥ | T | T | ⊥ | T | T |
| ⊥ | T | T | ⊥ | T | T | T | T | T |
| ⊥ | T | ⊥ | ⊥ | T | ⊥ | T | T | T |
| ⊥ | ⊥ | T | ⊥ | T | T | T | T | T |
| ⊥ | ⊥ | ⊥ | ⊥ | T | T | T | T | T |

## 5. 含意

例

(1) ${\sim}(P \to Q)$ は${\sim}Q$ を含意する.

| $P$ | $Q$ | $P \to Q$ | ${\sim}(P \to Q)$ | ${\sim}Q$ |
|---|---|---|---|---|
| T | T | T | ⊥ | ⊥ |
| T | ⊥ | ⊥ | T | T |
| ⊥ | T | T | ⊥ | ⊥ |
| ⊥ | ⊥ | T | ⊥ | T |

(2) $P \& Q$ は $P \vee Q$ を含意する.

| $P$ | $Q$ | $P \& Q$ | $P \vee Q$ |
|---|---|---|---|
| T | T | T | T |
| T | ⊥ | ⊥ | T |
| ⊥ | T | ⊥ | T |
| ⊥ | ⊥ | ⊥ | ⊥ |

## 6. 同値 (equivalence)

例

(1)

| $P$ | $Q$ | $P \& Q$ | ${\sim}(P \& Q)$ | ${\sim}P$ | ${\sim}Q$ | ${\sim}P \vee {\sim}Q$ |
|---|---|---|---|---|---|---|
| T | T | T | ⊥ | ⊥ | ⊥ | ⊥ |
| T | ⊥ | ⊥ | T | ⊥ | T | T |
| ⊥ | T | ⊥ | T | T | ⊥ | T |
| ⊥ | ⊥ | ⊥ | T | T | T | T |

(2)

| $P$ | $Q$ | $P \leftrightarrow Q$ | $P \& Q$ | ${\sim}P$ | ${\sim}Q$ | ${\sim}P \& {\sim}Q$ | $(P \& Q) \vee ({\sim}P \& {\sim}Q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | ⊥ | ⊥ | ⊥ | T |
| T | ⊥ | ⊥ | ⊥ | ⊥ | T | ⊥ | ⊥ |
| ⊥ | T | ⊥ | ⊥ | T | ⊥ | ⊥ | ⊥ |
| ⊥ | ⊥ | T | ⊥ | T | T | T | T |

(3)

| $P$ | $Q$ | $R$ | $Q \vee R$ | $P \& (Q \vee R)$ | $P \& Q$ | $P \& R$ | $(P \& Q) \vee (P \& R)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | ⊥ | T | T | T | ⊥ | T |
| T | ⊥ | T | T | T | ⊥ | T | T |
| T | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |
| ⊥ | T | T | T | ⊥ | ⊥ | ⊥ | ⊥ |
| ⊥ | T | ⊥ | T | ⊥ | ⊥ | ⊥ | ⊥ |
| ⊥ | ⊥ | T | T | ⊥ | ⊥ | ⊥ | ⊥ |
| ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

(4)

| $P$ | $Q$ | $R$ | $Q \& R$ | $P \vee (Q \& R)$ | $P \vee Q$ | $P \vee R$ | $(P \vee Q) \& (P \vee R)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | ⊥ | ⊥ | T | T | T | T |
| T | ⊥ | T | ⊥ | T | T | T | T |
| T | ⊥ | ⊥ | ⊥ | T | T | T | T |
| ⊥ | T | T | T | T | T | T | T |
| ⊥ | T | ⊥ | ⊥ | ⊥ | T | ⊥ | ⊥ |
| ⊥ | ⊥ | T | ⊥ | ⊥ | ⊥ | T | ⊥ |
| ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |

練習問題

(1) $\sim(P \vee Q)$, $\sim P \& \sim Q$［ド・モルガンの法則］

| $P$ | $Q$ | $P \vee Q$ | $\sim(P \vee Q)$ | $\sim P$ | $\sim Q$ | $\sim P \& \sim Q$ |
|---|---|---|---|---|---|---|
| T | T | T | ⊥ | ⊥ | ⊥ | ⊥ |
| T | ⊥ | T | ⊥ | ⊥ | T | ⊥ |
| ⊥ | T | T | ⊥ | T | ⊥ | ⊥ |
| ⊥ | ⊥ | ⊥ | T | T | T | T |

(2) $P \to Q$, $\sim Q \to \sim P$［対偶の法則］

| $P$ | $Q$ | $P \to Q$ | $\sim Q$ | $\sim P$ | $\sim Q \to \sim P$ |
|---|---|---|---|---|---|
| T | T | T | ⊥ | ⊥ | T |
| T | ⊥ | ⊥ | T | ⊥ | ⊥ |
| ⊥ | T | T | ⊥ | T | T |
| ⊥ | ⊥ | T | T | T | T |

(3) $P \to Q$, $\sim P \vee Q$

| $P$ | $Q$ | $P \to Q$ | $\sim P$ | $\sim P \vee Q$ |
|---|---|---|---|---|
| T | T | T | ⊥ | T |
| T | ⊥ | ⊥ | ⊥ | ⊥ |
| ⊥ | T | T | T | T |
| ⊥ | ⊥ | T | T | T |

(4) $P \to Q$, $\sim(P \& \sim Q)$

| $P$ | $Q$ | $P \to Q$ | $\sim Q$ | $P \& \sim Q$ | $\sim(P \& \sim Q)$ |
|---|---|---|---|---|---|
| T | T | T | ⊥ | ⊥ | T |
| T | ⊥ | ⊥ | T | T | ⊥ |
| ⊥ | T | T | ⊥ | ⊥ | T |
| ⊥ | ⊥ | T | T | ⊥ | T |

(5) $(P \& Q) \to R$, $P \to (Q \to R)$［移出律］

| $P$ | $Q$ | $R$ | $P \& Q$ | $(P \& Q) \to R$ | $Q \to R$ | $P \to (Q \to R)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | ⊥ | T | ⊥ | ⊥ | ⊥ |
| T | ⊥ | T | ⊥ | T | T | T |
| T | ⊥ | ⊥ | ⊥ | T | T | T |
| ⊥ | T | T | ⊥ | T | T | T |
| ⊥ | T | ⊥ | ⊥ | T | ⊥ | T |
| ⊥ | ⊥ | T | ⊥ | T | T | T |
| ⊥ | ⊥ | ⊥ | ⊥ | T | T | T |

(6) $\sim(P \leftrightarrow Q)$, $\sim P \leftrightarrow Q$

| $P$ | $Q$ | $P \leftrightarrow Q$ | $\sim(P \leftrightarrow Q)$ | $\sim P$ | $\sim P \leftrightarrow Q$ |
|---|---|---|---|---|---|
| T | T | T | ⊥ | ⊥ | ⊥ |
| T | ⊥ | ⊥ | T | ⊥ | T |
| ⊥ | T | ⊥ | T | T | T |
| ⊥ | ⊥ | T | ⊥ | T | ⊥ |

## 7. 妥当性 (validity)

練習問題 1

(1) $P \to Q, P \therefore Q$［肯定式 modus ponens］

| $P$ | $Q$ | $P \to Q$ | |
|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊥ | ⊥ | |
| ⊥ | ⊤ | ⊤ | |
| ⊥ | ⊥ | ⊤ | |

(2) $P \to Q, \sim Q \therefore \sim P$［否定式 modus tollens］

| $P$ | $Q$ | $P \to Q$ | $\sim Q$ | $\sim P$ | |
|---|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊥ | ⊥ | |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊥ | |
| ⊥ | ⊤ | ⊤ | ⊥ | ⊤ | |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ✓ |

(3) $P \to Q, Q \to R \therefore P \to R$［仮言三段論法］

| $P$ | $Q$ | $R$ | $P \to Q$ | $Q \to R$ | $P \to R$ | |
|---|---|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊤ | ⊥ | ⊤ | ⊥ | ⊥ | |
| ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊤ | |
| ⊤ | ⊥ | ⊥ | ⊥ | ⊤ | ⊥ | |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ✓ |

(4) $P \vee Q, \sim P \therefore Q$［否定肯定式 modus tollendo ponens］

| $P$ | $Q$ | $P \vee Q$ | $\sim P$ | |
|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊥ | |
| ⊤ | ⊥ | ⊤ | ⊥ | |
| ⊥ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊥ | ⊥ | ⊥ | ⊤ | |

(5) $P \vee Q, P \to R, Q \to R \therefore R$［単純構成的両刀論法］

| $P$ | $Q$ | $R$ | $P \vee Q$ | $P \to R$ | $Q \to R$ | |
|---|---|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊤ | ⊥ | ⊤ | ⊥ | ⊥ | |
| ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊥ | ⊤ | |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊥ | ⊤ | ⊥ | ⊤ | ⊤ | ⊥ | |
| ⊥ | ⊥ | ⊤ | ⊥ | ⊤ | ⊤ | |
| ⊥ | ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | |

練習問題 2

(1) $P \vee Q, Q \to R, \sim P \therefore Q \& R$

| | | | 仮定 1 | 仮定 2 | 仮定 3 | 結論 | |
|---|---|---|---|---|---|---|---|
| $P$ | $Q$ | $R$ | $P \vee Q$ | $Q \to R$ | $\sim P$ | $Q \& R$ | |
| ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | |
| ⊤ | ⊤ | ⊥ | ⊤ | ⊥ | ⊥ | ⊥ | |
| ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ⊥ | ⊥ | |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | |
| ⊥ | ⊥ | ⊤ | ⊥ | ⊤ | ⊤ | ⊥ | |
| ⊥ | ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | |

よってこの推論は妥当である.

(2) $P \to {\sim}(Q \,\&\, R), {\sim}P \lor R \therefore {\sim}Q \to P$

| | | | | | 仮定 1 | | 仮定 2 | | 結論 | |
|---|---|---|---|---|---|---|---|---|---|---|
| $P$ | $Q$ | $R$ | $Q \,\&\, R$ | ${\sim}(Q \,\&\, R)$ | $P \to {\sim}(Q \,\&\, R)$ | ${\sim}P$ | ${\sim}P \lor R$ | ${\sim}Q$ | ${\sim}Q \to P$ | |
| ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | ⊥ | ⊥ | ⊤ | ⊥ | ⊤ | |
| ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊥ | ⊤ | |
| ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | ✓ |
| ⊥ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | ✓ |
| ⊥ | ⊥ | ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | × |
| ⊥ | ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | × |

よってこの推論は妥当ではない.

(3) $R \to P, {\sim}Q \to R \therefore P \lor Q$

| | | | | 仮定 1 | 仮定 2 | 結論 | |
|---|---|---|---|---|---|---|---|
| $P$ | $Q$ | $R$ | ${\sim}Q$ | $R \to P$ | ${\sim}Q \to R$ | $P \lor Q$ | |
| ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊤ | |
| ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | |
| ⊥ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ✓ |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊤ | ⊥ | |
| ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | |

よってこの推論は妥当である.